



1. Integers, divisors, multiples

E.8244 Complete the table with crosses to indicate whether the integers shown are divisible by 2, 3, 5, 9.

Entiers	123	504	205	1433	2430
Divisible by 2					
Divisible by 3					
Divisible by 5					
Divisible by 9					

E.8245

① Give the expression of the fractions below in irreducible form:

(a) $\frac{14}{26}$ (b) $\frac{66}{27}$ (c) $\frac{15}{55}$ (d) $\frac{56}{40}$

② Perform the operations below, simplifying each of the calculation terms beforehand:

(a) $\frac{15}{25} + \frac{9}{15}$ (b) $\frac{42}{14} - \frac{36}{12}$

E.8243

L the integer 10 has 4 factors which are:
1 ; 2 ; 5 ; 10

Complete the table below:

Entier x	2	3	4	5	6	7	8	9	10	11	12
Nombre de diviseurs de x									4		

E.8250 Which of the numbers below admits exactly 5 factors:

10 25 35 81 125

E.8251 Which of the numbers below admits exactly 4 factors:

24 28 49 64 343

E.10545 I am a number divisible by 5 and by 7, I am smaller than 100 and I have 8 factors. Who am I?

E.8252 I am a number divisible by 6 and by 21, I am smaller than 100 and I have 8 factors. Who am I?

E.8246 In this exercise, any trace of research, however incomplete, or initiative, however unsuccessful, will be taken into account in the assessment.

“The hidden number :

- I am an integer between 100 and 400.
- I am even.
- I am divisible by 11.
- I also have 3 and 5 as factors.

Qui suis-je?”.

Explain a procedure for finding the hidden number, and give its value.

E.11744

① Compléter les pointillés:

$$\begin{aligned} 13 &= \dots \times 3 + 1 & 17 &= \dots \times 3 + 2 \\ 14 &= 4 \times 3 + \dots & 18 &= 6 \times 3 + \dots \\ 15 &= \dots \times 3 & 19 &= \dots \times 3 + 1 \\ 16 &= 5 \times 3 + \dots & 20 &= 6 \times 3 + \dots \end{aligned}$$

② Que peut-on dire de l'écriture des nombres multiples de 3?

2. Odd and even integers

E.1771

① (a) Perform the Euclidean divisions below (and not the decimal divisions) and complete the relationship shown under each:

15	2	28	2	131	2	206	2

(b) Complete the following relationships related to question (a):

$$\begin{aligned} \bullet 15 &= \dots \times 2 + \dots & \bullet 28 &= \dots \times 2 + \dots \\ \bullet 131 &= \dots \times 2 + \dots & \bullet 206 &= \dots \times 2 + \dots \end{aligned}$$

② What can be said about the remainder of the Euclidean division by 2 of an even number? the remainder of the Euclidean division by 2 of an odd number?

E.8261    Complete the following two double-entry tables:

+	Pair	Impair
Pair		
Impair		

×	Pair	Impair
Pair		
Impair		

E.8259    Complete the following sentences without justification using the words **even**, **odd**, **any**.

- The sum of two even integers is an integer
- The sum of two odd integers is an integer
- The product of two odd integers is an integer
- The product of an even integer by an odd integer is an integer

E.8267    Without justification, say whether the following assertions are true, false or undecidable:

- The sum of two odd integers is an even integer.

3. Parity and algebraic manipulations

E.8256   

- Let k and k' be two relative integers ($k, k' \in \mathbb{Z}$), expand and reduce the expression: $(2 \cdot k + 1)(2 \cdot k' + 1)$
 - Deduce the parity of the product of two odd integers.

- Deduce the parity of the square of an odd number.

E.3027    Show that the sum of four consecutive integers is an even integer.

E.9468    Show that, for any odd integer n , the expression:

$$3 \cdot n^2 + 2 \cdot n + 1$$

defines an even integer.

E.9469    Show that, for any integer n , the expression:

$$n^2 + 3 \cdot n$$

is an even integer.

E.233    In this exercise, we will use the fact that any even natural number (*resp.* odd) is written as $2 \times n$ (*resp.* $2 \times n + 1$) where n is a natural number.

Prove the following assertions:

- The sum of two odd integers is an even integer.
- The product of an even integer and an odd integer is even.
- The product of two consecutive integers is an even integer.
- The sum of five consecutive integers is a multiple of 5.

E.11259    Consider the integer A defined by:

$$A = (n + 1)^2 - (n - 1)^2 \quad \text{where } n \in \mathbb{N}.$$

Show that, for any integer n , the integer A is a multiple of 4.

- The product of an even integer and an odd integer is even.
- The product of two consecutive integers is an even integer.
- The sum of five consecutive integers is a multiple of 5.

E.8361   

Reminder: parity of operations:

+	Even	Odd
Even	Even	Odd
Odd	Odd	Even

×	Even	Odd
Even	Even	Even
Odd	Even	Odd

Let a be an integer such that the integer $a^2 + 9$ is an even number.

What can be said about the parity of the integer a ? Justify your answer.

E.11291   

- Copy and complete the dotted lines so that the expression on the left is equal to the expression on the right:

$$(2 \cdot k + 1)^2 = 2(\dots\dots\dots) + 1$$

- The identity in the previous question allows us to state:

"The square of is an integer"

Copy and complete the sentence above.

E.11445    Let a and b be two odd integers. Show that the integer $a^2 + b^2 + 6$ is a multiple of 8.

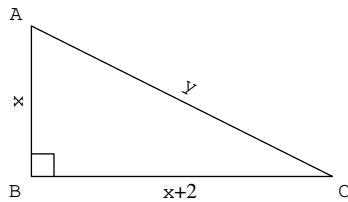
E.11257    Show that the integer preceding the square of any odd integer is a multiple of 4.

Hint: The goal of this exercise is to generalize the following observation:

$$3^2 - 1 = 8 = 2 \times 4 \quad ; \quad 5^2 - 1 = 24 = 6 \times 4 \quad ; \quad 7^2 - 1 = 48 = 12 \times 4$$

$$9^2 - 1 = 80 = 20 \times 4 \quad ; \quad 11^2 - 1 = 120 = 30 \times 4 \quad ; \quad 13^2 - 1 = 168 = 42 \times 4$$

E.9479 🔧 🍷 📖 Consider the triangle ABC right-angled B , shown below, such that $BC=AB+2$ and its measures are integers:



We model the situation by noting $AB=x$ and $AC=y$.

- 1 (a) Express y^2 in terms of x as an expanded and simplified expression.
- (b) Deduce that the integer y is even.
- 2 (a) Justify that $2 \cdot x^2 + 4 \cdot x + 4$ is a multiple of 4.
- (b) Deduce that the integer x is even.
- 3 Complete the algorithm below giving us the values of x and y (with $y < 1000$) realizing the dimensions of this triangle:

```
import math
for x in range(...):
    y=math.sqrt(...)
```

```
if math.floor(...)==...:
    print(x,y)
```

E.8260 🔧 🍷 📖 ⚠️

- 1 Let M be a natural number, odd and not prime. Suppose that $M=a^2-b^2$ where a and b are two natural numbers.
Show that a and b do not have the same parity.
- 2 Let N be an integer that can be factored:
 $N=p \times q$
We assume that this factorization is related to two integers a and b allowing the identity:
 $p=(a+b)$; $q=(a-b)$
 - (a) Express the integers a and b in terms of p and q .
 - (b) Justify that the integers p and q have the same parity.
 - (c) Deduce the representation of the integer N as a difference of squares, in terms of p and q .
- 3 Write the integer 63 in three different forms as a subtraction of squares.

E.9470 🔧 🍷 📖 Show that for any integer k , the integer k^2+k is even.

4. Prime numbers

E.1721 🔧 🍷 📖 The Eratosthenes sieve (III^{ième} century BC) makes it easy to find prime integers from a list.

- 1 (a) Justify that 2 is a prime integer.
- (b) In the table below, hatch all the boxes whose integer is a multiple of 2 (do not hatch the box "2").
- 2 (a) Justify that 3 is a prime integer.
- (b) In the table below, hatch all boxes whose integer is a multiple of 3 (do not hatch the box "3").
- 3 (a) The white square following the square for 3 is the square for 5: justify, using this observation, that 5 is a prime integer.
- (b) Hatch all squares that are multiples of 5, except the square "5".
- 4 (a) In the table below, what is the white box succeeding "5"?
Also justify, using the observation of the table, that 7 is a prime integer.
- (b) Hatch all the multiples of the integer 7 in the table (except the box "7").
- 5 Continue work to hatch in the table all non-premier integers present among the integers from 1 to 100.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

E.8253   

Definition:

- The set of positive or zero integers is called the **set of natural integers** and is denoted $\mathbb{N}$.
Thus: $\mathbb{N} = \{0; 1; 2; 3; 4; \dots\}$
- A natural integer n ($n \in \mathbb{N}$) is said to be a **integer premier** if it admits only two factors 1 and itself.
 - ▶ The integer 7 is a prime integer because it admits as factor: 1 and 7.
 - ▶ The integer 9 is not a prime integer because it admits 3 factors: 1, 3 and 9.

- 1 Justify that the numbers 32 and 63 are not prime integers.
- 2 Justify that the number 17 is a prime integer.

E.1722    A natural number is said to be prime

if it admits as divisors only 1 and itself: 3 is a prime number, because its only divisors are 1 and 3.

- 1 Which of the following integers are prime?
2 ; 4 ; 7 ; 12

- 2 Give all prime integers from 1 to 25.

E.8248   

- 1 Name the ten prime integers less than or equal to 30.
- 2 Which of the numbers below are prime numbers.

33 47 51 28 39 49 85

E.8255    Justify that each of the sentences below is a false assertion:

- 1 The sum of two prime integers is a prime integer.
- 2 The difference of two prime integers is a prime integer.
- 3 The product of two prime integers is a prime integer.

5. Decomposition into product of prime factors

E.240    We're going to study the algorithm for decomposing integers into products of prime factors.

- 1 Name the eight prime integers less than 20.

The table opposite shows the algorithm for decomposing 30 into a product of prime factors:

30	2	$30 \div 2 = 15$
15	3	$15 \div 3 = 5$
5	5	$5 \div 5 = 1$
1		

- ▶ The left-hand column represents the integer 30 and the successive quotients obtained.
- ▶ The column on the right represents the divisors used; note that only prime integers are used.

The algorithm stops when 1 is obtained in the left-hand column.

- 2 a Justify, in view of the above table, the following equality:

$$\frac{30}{2} = \frac{15}{3} = \frac{5}{5} = 1$$

- b Let x be a real number and a , b and c be three non-zero real numbers, establish the following equality:

$$\frac{\frac{x}{a}}{\frac{b}{c}} = \frac{x}{a \times b \times c}$$

- c Deduce from the previous questions, that the prime factor decomposition of 30 is:

$$30 = 2 \times 3 \times 5$$

E.7122   

- 1 Use the previous algorithm to determine the prime factor decomposition of the following integers:

a 84 b 144 c 140 d 196

- 2 Deduce the product of prime factors decomposition of the following products:

a 84×144 b 140×196

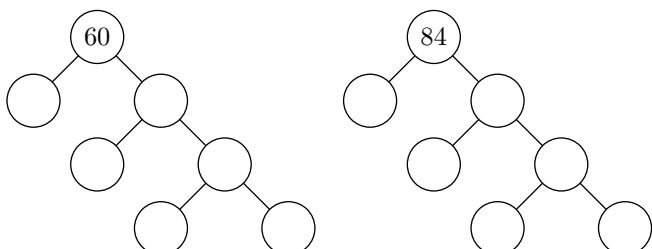
E.255    Give the product of prime factors decomposition of the three integers below:

a 16×25 b 34×12 c $72 \times 18 \times 10$ d 32×121

6. Product of prime factors, divisors and multiples

E.11288   

- 1 Give the prime factorization of the integers 60 and 84.



- 2 Deduce the prime factorization of 5040.

Hint, Note that: $60 \times 84 = 5040$

E.9578    Here is the prime factor decomposition of the integer 360: $360 = 2^3 \times 3^2 \times 5$

- 1 Justify that the following integers are factors of 360:
- (a) $2^1 \times 3^2$ (b) 2×5 (c) $2^2 \times 3^0 \times 5$

- 2 Which of the following integers are factors of 360:

- (A) $2^0 \times 3^0 \times 5^0$ (b) $2^4 \times 3$
(c) 2×5^2 (d) $2^3 \times 3^2 \times 5$

- 3 What conditions, on the exponents, can be given to the integer $2^n \times 3^n \times 5^p$ for it to be a factor of 360?

7. Product of prime factors and reduction of fractions

E.8424    1 Give the prime factor product decomposition of the numbers 60 and 450.

- 2 Deduce the simplified expression for the quotient $\frac{60}{450}$

- 3 Perform the sum: $\frac{1}{60} + \frac{1}{450}$

E.234    1 Give the decomposition of the integers 108 and 30 into products of prime factors.

- 2 Highlight your approach to the following two questions:

- (a) Simplify the fraction $\frac{30}{108}$.
(b) Perform the subtraction below and give the result as an irreducible fraction:
$$\frac{108}{5} - \frac{7}{30}$$

E.1805    1 Give the decomposition into products of prime factors of the integers 20 and 135.

- 2 Answer the questions below, justifying the approach or indicating the steps in the calculations:

- (a) Simplify the fraction $\frac{20}{135}$.
(b) Perform the following subtraction: $\frac{7}{20} - \frac{8}{135}$

E.9529    1 Give the prime factor product decomposition of the integer 36.

- 2 The following prime product decomposition is given: $56 = 2^3 \times 7$

- (a) Give the greatest common factor to 36 and 56.
(b) Deduce the expression reduce from the expression $\frac{36}{56}$

- 3 The following decomposition into products of prime factors is given: $42 = 2 \times 3 \times 7$

- (a) Give the values of the natural numbers a , b , c such that the integer $n = 2^a \times 3^b \times 7^c$ is the smallest multiple of 36 and 42.
(b) Deduct the sum of: $\frac{1}{36} + \frac{1}{42}$

E.3361    1 Give the product of prime factors decomposition of the following integers:

- (a) 2016 (b) 2100 (c) 864

- 2 Perform the following operations and give the result in simplified form:

- (a) $\frac{2016}{2100}$ (b) $\frac{1}{2100} + \frac{1}{864}$

8. Powers

E.7123    1 Give the prime factor product decomposition of the integers: 16 ; 24

- 2 (a) Determine the prime factor decomposition of the product: 16×24 .

- (b) Determine the product decomposition of prime factors of the product: $16^3 \times 24^2$.

- 3 Determine the prime factor product decomposition of the quotient: $\frac{24^5}{16^2}$.

E.9466    Simplify the following integers as products of prime factors:

- (a) $18 \times 15^2 \times 9 \times 82$ (b) $9^2 \times 15^{-2} \times 4^4$

E.3362    Simplify the following integers as products of prime factors:

- (a) $\frac{5 \times 3^4 \times 12^2}{21^2 \times 15^3}$

E.26    1 Indicate whether the following numbers are cubes of natural numbers:

- (a) 9 (b) 27
(c) $3^3 \times 7^3$ (d) $11^9 \times 19^6 \times 67^3$ (e) $2^2 \times 5^3 \times 11$

- 2 (a) Find the number b such that $180 \times b$ is the cube of a natural number

- (b) Is this number b unique?

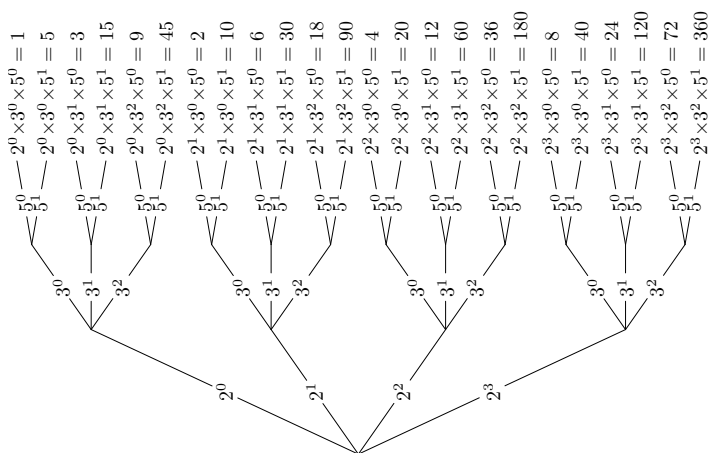
E.235 🔑 📁 Using the product decomposition of prime factors, write the numbers below in the form:

$2^m \times 3^n \times \dots$ where exponents are integers **relative**.

- (a) $\frac{9}{24}$ (b) $\frac{28^2}{32}$ (c) $\frac{81 \times 6^4}{27^2 \times 7^5}$ (d) $\frac{38^2 \times 11}{6^5 \times 4^4 \times 19}$

9. Integers and numbers of divisors

E.260 🔑 📁 The decision tree below gives all the 24 divisors of the integer 360.



Using this example, determine the set of divisors of the following integers:

- (a) 12 (b) 135

E.9467 🔑 📁 Simplify the following quotients as products of prime factors:

(a) $\frac{5^2 \times 3^2}{5^5 \cdot (3^4 + 3^4)}$ (b) $\frac{2^2 \times 5^{-4}}{2^{21} + 2^{22}}$

E.9530 🔑 📁 Consider the integer: $a = 156$

- Give the product decomposition of prime factors of the number a .
- (a) Justify that $2^2 \times 17$ cannot be a factor of a .
(b) Justify that the integer $2^2 \times 3^2$ is not a factor of a .
- Give the set of factors of the integer a .

Reminder: the prime integers less than 20 are:

2 ; 3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19

E.276 🔑 📁

- Determine the decomposition of 245 into a product of prime factors.
- Use a choice tree, to determine the set of divisors of 245.

E.9536 🔑 📁

- Give the prime factor product decomposition of the integer 306.
- Give the set of factors of the integer 306.

10. In depth: parity and reasoning by absurdity

E.8257 🔑 📁

- Prove that the following statement is true:
"There are no odd integers that have an even divisor"

Hint: To prove this statement, assume that there is an integer n that has an even divisor k . It then suffices to show that this assumption leads to a contradiction: this assumption cannot therefore be true.

- What do you think of the statement:
"An even integer does not have an odd divisor"?

11. Deepening: parity and reasoning by disjunction of cases

E.8360 🔑 📁 Prove that the sum of two integers with the same parity is an even integer.

Hint: for this, we'll study separately:

- the sum of two even integers
- and the sum of two odd integers.

E.9532 🔑 📁 Show that the difference between the squares of two consecutive integers is always an odd integer.

E.1847 🔑 📁 We want to show that the expression $A = n^2 + n + 2$ defines an even number for every natural number n ($n \in \mathbb{N}$).

To do this, we break the argument down into two questions:

- When n is an even integer, show that the expression A defines an even integer.
- When n is an odd integer, show that the expression A defines an even integer.

E.8263 🔥 🍷 📖 Consider a relative integer a ($a \in \mathbb{Z}$), such that the integer a^2+9 is an even integer. Give the parity of the integer a .

E.65 🔥 🍷 📖 Show that $3n^2+3n+6$ is a multiple of 6 for all $n \in \mathbb{N}$

E.8262 🔥 🍷 📖 Show that, in the integer set of relative integers, the predecessor of the square of any odd integer is a multiple of 8.

Hint: initially, we'll show that this integer is a multiple of 4, then by a disjunction of cases, we'll show that it is necessarily a multiple of 8.

12. In depth: primality criterion

E.8247 🔥 🍷 📖

The following two propositions are accepted:

Proposition (admitted):

Let n be a natural number ($n \in \mathbb{N}$). If n is not a prime integer then there exists at least one prime integer p factor of n such that p is between 2 and $\sqrt{n}$.

Contradictory proposition:

Let n be a natural number ($n \in \mathbb{N}$). If the integer n admits no prime factor p between 2 and $\sqrt{n}$, then the integer n is prime.

1 Let's use the **proposition**:

- a Name all prime integers between 2 and $\sqrt{143}$.
- b Justify that the number 143 is not a prime integer.

2 Using the **contraposé proposition**:

- a Name all prime integers between 2 and $\sqrt{157}$.
- b Justify that the number 157 is a prime integer.

3 In the list of numbers below, cross out the numbers that are not prime integers:

93 119 253 383 399 631

To see the demonstration of the "proposition admitted", suivez le lien ci-contre



E.1783 🔥 🍷 📖 Determine whether the following integers are prime or not. Explain your reasoning.

E.8258 🔥 🍷 📖

1 For any relative integer n ($n \in \mathbb{Z}$), consider the expression: $p=2 \cdot n^2+3 \cdot n+3$

- a If n is even, show that the expression p defines an odd integer.

Hint: an even integer n is written $2 \cdot k$ where $k \in \mathbb{Z}$

- b Show that, if n is odd, the expression p defines an even integer.

2 Justify that the expression n^2+n+3 defines an odd integer for any relative integer n .

- a 251
- b 623

E.8254 🔥 🍷 📖 ⚠ Among the integers strictly less than 1 000, give the largest prime integer.

E.1727 🔥 🍷 📖 The aim of this exercise is to establish the following proposition:

Proposition:

Any natural integer n non-premier admits a prime divisor lying in the interval $[2; \sqrt{n}]$.

To do this, let's take a non-first natural number x ; then there are two natural numbers a and b greater than or equal to 2 such that:

$$x = a \times b$$

Let's assume that a is smaller than b (this is possible by reversing the role of "a" and "b" if necessary)

To show that a belongs to the interval $[2; \sqrt{x}]$ we'll reason by the absurd.

To do this, let's assume that the integer belongs to the interval $] \sqrt{x}; +\infty[$

- 1 Justify the following inequality: $a \times b > x$
- 2 Infer that: $a \in [2; \sqrt{x}]$.

The demonstration ends from the fact that a is a divisor of x belonging to the interval $[2; \sqrt{x}]$:

- either a is prime;
- or a is not prime and a then admits prime divisors; these divisors will also be prime divisors of x belonging to the interval under consideration.

13. Greatest common divisor: decomposition into product of prime factors

E.10653 🔥 🍷 📖

Method: let a and b be two integers whose decomposition into products of prime factors is known.

The prime factor decomposition of $\text{PGCD}(a,b)$ verifies:

- its factors are the factors common to each of the decompositions of a and b .
- the exponent of each of its factors is the smallest exponent of that factor encountered in the decompositions of a and b .

Example: for $a = 36$ and $b = 60$, we have:

$$a = 2^2 \times 3^2 ; \quad b = 2^2 \times 3 \times 5$$

Thus, the decomposition of $PGCD(36,60)$ verifies:

- its factors are 2 and 3.
- The exponent of the 2 factor is 2; the exponent of the 3 factor is 1.

On en déduit la valeur de:

$$PGCD(36,60) = 2^2 \times 3^1 = 4 \times 3 = 12$$

- 1 a Determine the prime factor product decomposition of 126.
- b Determine the prime factor product decomposition of 108.
- 2 Deduct the value $PGCD(126,108)$.

E.10693   

- 1 Determine the prime factor product decomposition of the integers 168 and 192.

- 2 Deduct the PGCD of the integers 168 and 192.

E.10694   

- 1 Determine the prime factor product decomposition of the integers 162 and 378.

- 2 Deduct the PGCD of the integers 162 and 378.

E.10700   

- 1 Determine the prime factor product decomposition of the integers 171 and 228.

- 2 Deduct the PGCD of the integers 171 and 228.

E.10576   

- 1 Determine the prime factorizations of the integers 756 and 792.

- 2 Deduct the GCD of the integers 756 and 792.

14. Least common multiple with decomposition

E.10662   

Method: let a and b be two integers whose decomposition into products of prime factors is known.

The decomposition of $PPCM(a,b)$ has the following characteristics:

- the set of factors contained in the decompositions of a and b
- the exponents of each of these factors is the maximum exponent encountered in the decompositions of a and b .

Example: for $12 = 2^2 \times 3$ et $15 = 3 \times 5$.

- the $PPCM(a,b)$ has as factor 2, 3 and 5.
- the $PPCM(a,b)$ recovers the largest exponents of the decomposition of 12 and 15.

On a: $PPCM(12,15) = 2^2 \times 3^1 \times 5^1 = 4 \times 3 \times 5 = 60$.

- 1 Determine the prime factor product decomposition of 20

and 75.

- 2 Deduct the value of $PPCM(20,75)$.

E.10895   

- 1 Determine the prime factor product decompositions of the integers 252 and 336.

- 2 Deduct the value of the PPCM of the integers 252 and 336.

E.10896   

- 1 Determine the prime factorization of the integers 336 and 840.

- 2 Deduct the least common multiple of the integers 336 and 840.

E.10897   

- 1 Determine the prime factor product decomposition of the integers 448 and 560.

- 2 Deduct the least common multiple to the integers 448 and 560.

15. Unclassified exercises

E.9584   Among the following integers, say whether they are prime or not. Justify your answers:

A 903

B 167