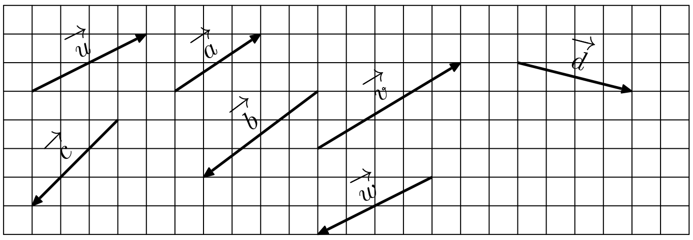
ChingQuizz : 9 exercises available for Quizz assessment :

1. *Opposite vectors and subtractions*

E.8530

Definition : Let $\vec{u}$ be a vector. The opposite vector of vector $\vec{u}$ is the vector denoted by $-\vec{u}$, defined as :
- the same direction as vector $\vec{u}$
- the opposite direction to vector $\vec{u}$
- the same length as $\vec{u}$

In the plane, consider the 7 vectors below :



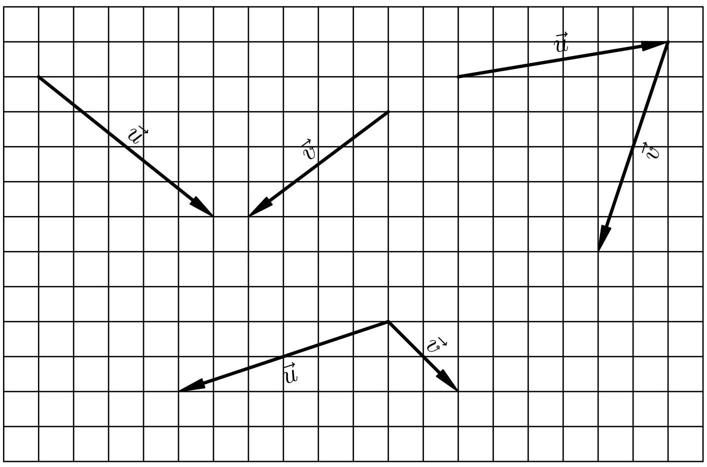
1. Name the vector(s) opposite to vector $\vec{u}$.
2. Draw a vector $\vec{e}$ opposite to vector $\vec{d}$.

E.524

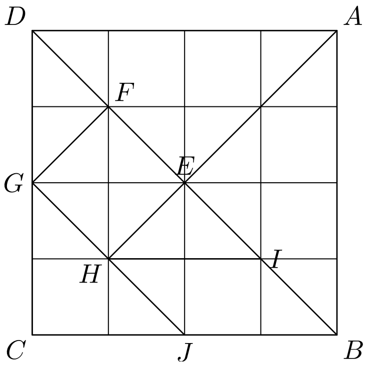
Definition : Let $\vec{u}$ and $\vec{v}$ be two vectors. We define the subtraction of vector $\vec{u}$ by vector $\vec{v}$ by the relation :
$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$
In other words, we never subtract a vector, we add its opposite

Let $\vec{u}$ and $\vec{v}$ be two vectors in the plane.

1. What can we say about the difference : $\vec{u} - \vec{u}$?
2. For each of the three cases shown below, draw a representation of the subtraction : $\vec{u} - \vec{v}$



E.495



Determine a representative of each of the sums below :
1. $\overrightarrow{EI} - \overrightarrow{GF}$
2. $\overrightarrow{HE} + \overrightarrow{BI} - \overrightarrow{JF}$
3. $\overrightarrow{FG} - \overrightarrow{IF} - \overrightarrow{GE}$

E.9342

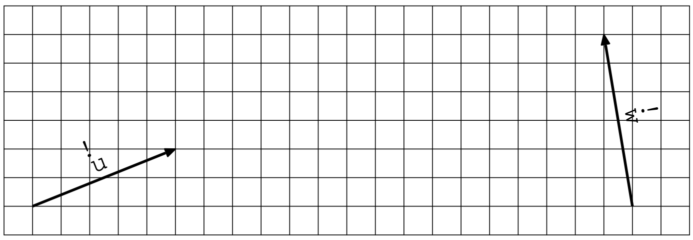
Let $ABCD$ be a parallelogram. Note :
- I the middle of segment $[AB]$;
- J the middle of segment $[DC]$.

Determine in each case a representative of the resulting vector :

a. $\overrightarrow{AC} - \overrightarrow{AJ}$ b. $\overrightarrow{AB} + \overrightarrow{IJ} - \overrightarrow{DJ}$ c. $-\overrightarrow{DJ} - \overrightarrow{JB}$

E.8531

In the plane, consider the two vectors $\vec{u}$ and $\vec{w}$ shown below.



Draw the vector $\vec{v}$ realizing the relation :
$$\vec{u} - \vec{v} = \vec{w}$$

E.6488

The plane is given a reference frame $(O\,;\ \vec{i}\ ;\ \vec{j})$ and the three points :
$$A\left(1+\sqrt{2}\,;2-\sqrt{2}\right)\ ;\ B\left(\sqrt{2}-1\,;\sqrt{2}+1\right)\ ;\ C\left(\sqrt{2}+3\,;3-3\sqrt{2}\right)$$

1. Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite.
2. What can be said about the point A relative to the segment $[BC]$.

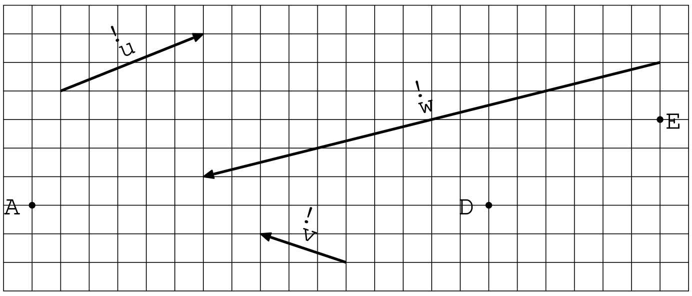
2. *Multiplication by an integer*

E.9713

Proposition : in the plane, consider a vector $\vec{u}$ and an integer $n \in \mathbb{N}^*$. We define the vector $n \cdot \vec{u}$ by :

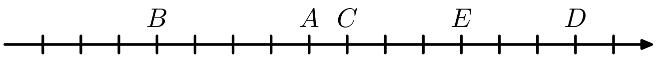
$$n \cdot \vec{u} = \underbrace{\vec{u} + \vec{u} + \cdots + \vec{u}}_{n \text{ fois}}$$

In the plane, consider the three vectors and three points shown below :



1. Place the point B such that : $\overrightarrow{AB} = 3 \cdot \vec{u}$
2. Place the point C such that : $\overrightarrow{CD} = 2 \cdot \vec{v}$
3. Place the point F such that : $\vec{w} = 4 \cdot \overrightarrow{EF}$

E.515 On a graduated line, are placed the points A, B, C, D, E :



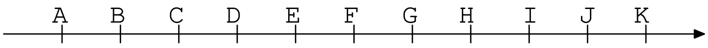
For each question, complete the dotted lines correctly :

a) $\overrightarrow{BC} = \ldots\ldots \times \overrightarrow{AC}$ b) $\overrightarrow{ED} = \ldots\ldots \times \overrightarrow{AC}$

c) $\overrightarrow{AC} = \ldots\ldots \times \overrightarrow{CA}$ d) $\overrightarrow{ED} = \ldots\ldots \times \overrightarrow{CA}$

e) $\overrightarrow{EA} = \ldots\ldots \times \overrightarrow{AB}$ f) $\overrightarrow{BA} = \ldots\ldots \times \overrightarrow{BE}$

E.9716 The drawing below shows a straight line with a regular scale.

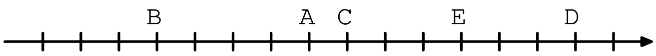


Complete the blanks with the missing number :

a) $\overrightarrow{DG} = \ldots\ldots\ \overrightarrow{DE}$ b) $\overrightarrow{CE} = \ldots\ldots\ \overrightarrow{GI}$

c) $\overrightarrow{DB} = \ldots\ldots\ \overrightarrow{DF}$ d) $\overrightarrow{EI} = \ldots\ldots\ \overrightarrow{AC}$

E.5287 On a graduated line, we place the points A, B, C, D, E :



For each question, determine the value of the number k verifying the equality :

a) $\overrightarrow{BC} = k \cdot \overrightarrow{AC}$ b) $\overrightarrow{ED} = k \cdot \overrightarrow{AC}$

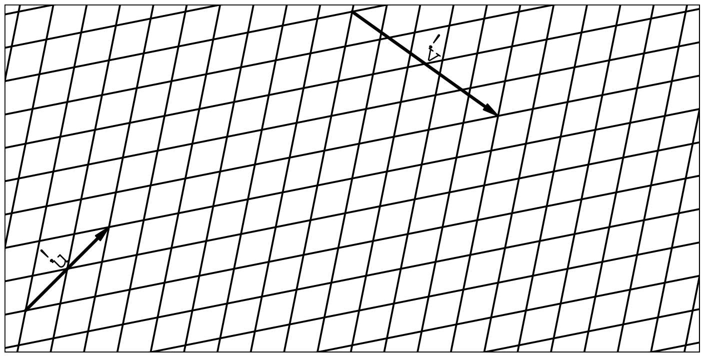
c) $\overrightarrow{AC} = k \cdot \overrightarrow{CA}$ d) $\overrightarrow{ED} = k \cdot \overrightarrow{CA}$

e) $\overrightarrow{EA} = k \cdot \overrightarrow{AB}$ f) $\overrightarrow{AC} = k \cdot \overrightarrow{BA}$

E.9811 Let A, B, C be three non-aligned plane points :

Simplify the expression : $2 \cdot \overrightarrow{AB} - \overrightarrow{BA}$

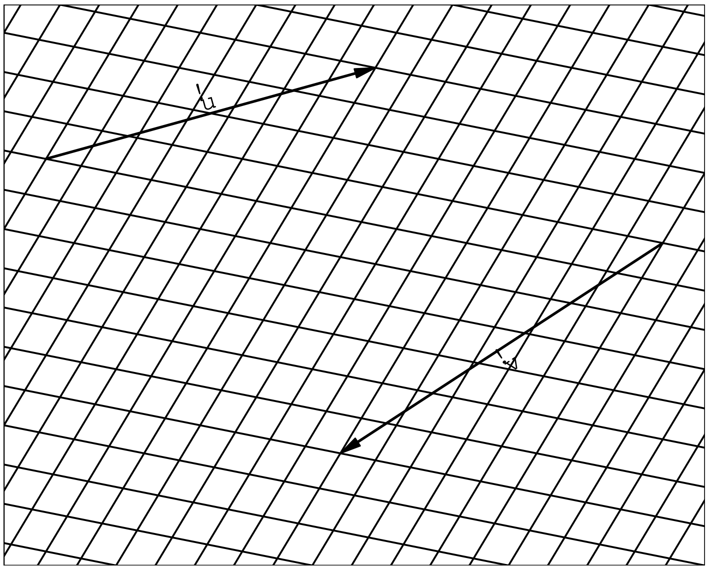
E.2077 Consider, in the plane, the two vectors $\vec{u}$ and $\vec{v}$ below :



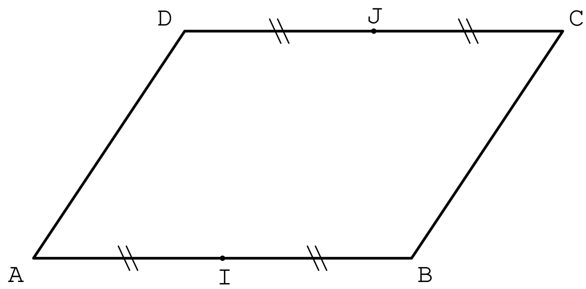
Draw in the grid a representative of the vector $\vec{w}$ defined by : $\vec{w} = 2\,\vec{u} + 3\,\vec{v}$.

E.2106 Consider the plane below, with a regular grid. Let $\vec{u}$ and $\vec{v}$ be two vectors of the plane :

1. Draw a representative $\vec{w}$ of the vector $\vec{v} - \vec{u}$.
2. Draw a representative $\vec{x}$ of the vector $4\,\vec{u} + 3\,\vec{v}$.



E.4813 Consider the parallelogram $ABCD$ shown below où the points I and J are the respective middles of the segments $[AB]$ and $[CD]$.



For each question, give without justification a vector equal to the proposed expression :

a) $2 \times \overrightarrow{DJ} + \overrightarrow{BD}$ b) $3 \cdot \overrightarrow{DJ} + 2 \cdot \overrightarrow{IA}$ c) $2 \cdot \overrightarrow{AJ} - \overrightarrow{BC}$

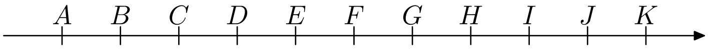
3. *Multiplications by a real*

E.523

Definition : In the plane, consider a vector $\overrightarrow{u}$ and a number $k\in\mathbb{R}$. The vectors $\overrightarrow{u}$ and $k\cdot\overrightarrow{u}$ are collinear, and :

	$\overrightarrow{u}$ et $k\cdot\overrightarrow{u}$	$\|\overrightarrow{v}\|$
$k<0$	opposite direction	$-k\times\|\overrightarrow{u}\|$
$k=0$	$\times$	0
$k>0$	same direction	$k\times\|\overrightarrow{u}\|$

The figure below shows a line with evenly spaced markings.



Fill in the dotted line with the missing number :

a) $\overrightarrow{EF}=\ldots\ldots\,\overrightarrow{GJ}$ b) $\overrightarrow{BD}=\ldots\ldots\,\overrightarrow{CF}$

c) $\overrightarrow{JI}=\ldots\ldots\,\overrightarrow{AC}$ d) $\overrightarrow{EF}=\ldots\ldots\,\overrightarrow{JG}$

e) $\overrightarrow{AE}=\ldots\ldots\,\overrightarrow{FC}$ f) $\overrightarrow{CG}=\ldots\ldots\,\overrightarrow{KA}$

E.484 Let A and B be two points of the plane, and let I be the midpoint of the segment $[AB]$

1. Complete the dotted lines to verify the following vector relationship : $\overrightarrow{AI}+\overrightarrow{AI}=\overrightarrow{A\ldots\ldots}$
2. Copy and complete with the words "*double*" and "*moitié*" the following sentences :

   a) $\overrightarrow{AI}$ est ... de $\overrightarrow{AB}$ b) $\overrightarrow{AB}$ est ... de $\overrightarrow{AI}$
3. In relation to the previous question, complete the dotted lines with the appropriate number :

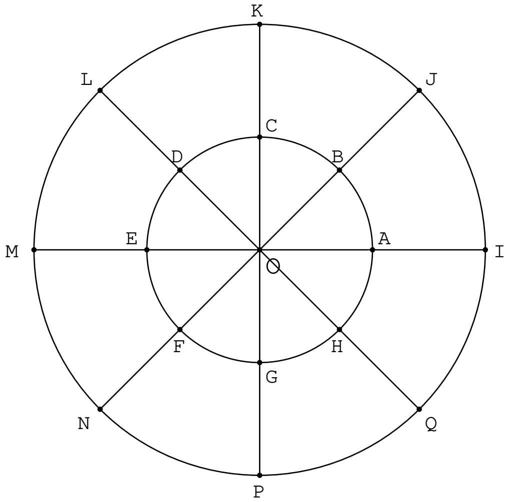
   a) $\overrightarrow{AI}=\ldots\ldots\,\overrightarrow{AB}$ b) $\overrightarrow{AB}=\ldots\ldots\,\overrightarrow{AI}$

E.485 Let ABC be any triangle. Place the points D and E verifying the following vector relations :

$$\overrightarrow{AD}=2\cdot\overrightarrow{AB}\quad;\quad\overrightarrow{AE}=2\cdot\overrightarrow{AC}$$

Compare $\overrightarrow{BC}$ and $\overrightarrow{DE}$. Justify.

E.6544 Consider the two concentric circles of center O and whose radius is twice that of the other :



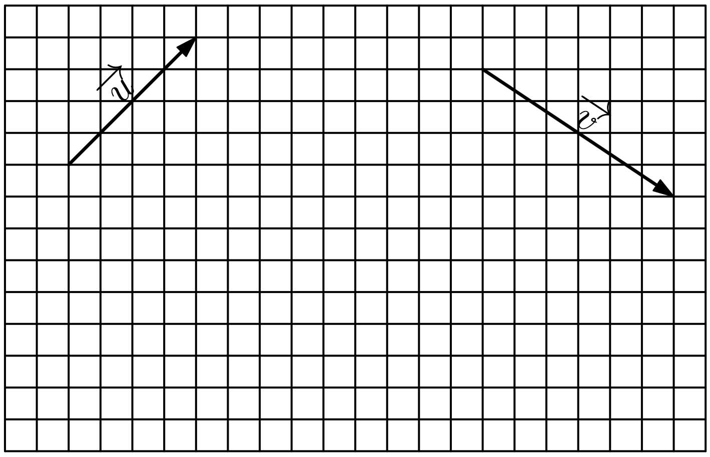
1. Justify vector equality : $\overrightarrow{LJ}=2\cdot\overrightarrow{DB}$
2. Without justification, complete the equalities :

   a) $\overrightarrow{ED}=\overrightarrow{\ldots\ldots}=\dfrac{1}{2}\cdot\overrightarrow{\ldots\ldots}=\dfrac{1}{2}\cdot\overrightarrow{\ldots\ldots}$

   b) $\overrightarrow{FB}=2\cdot\overrightarrow{\ldots\ldots}=2\cdot\overrightarrow{\ldots\ldots}=\dfrac{1}{2}\cdot\overrightarrow{\ldots\ldots}$

E.509 Consider below the two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$:

1. Draw a vector $\overrightarrow{w}$ representing from : $3\cdot\overrightarrow{u}+1.5\cdot\overrightarrow{v}$
2. Draw a vector $\overrightarrow{y}$ representing from : $\overrightarrow{v}-1.5\cdot\overrightarrow{u}$



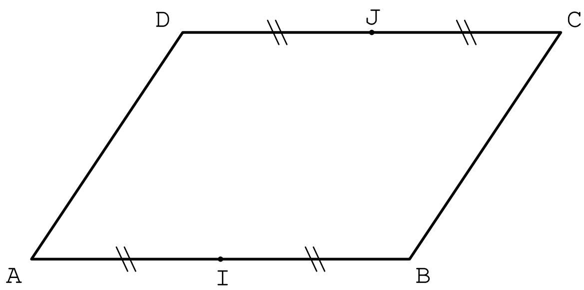
4. *Simplification and algebraic manipulation*

E.9809 In the plane, consider the three points A, B, C and define the two vectors $\overrightarrow{r}$ and $\overrightarrow{s}$ by :

$$\overrightarrow{r}=5\cdot\overrightarrow{AC}-2\cdot\overrightarrow{BC}\quad;\quad\overrightarrow{s}=2\cdot\overrightarrow{AB}+3\cdot\overrightarrow{AC}$$

Show that the two vectors $\overrightarrow{r}$ and $\overrightarrow{s}$ are equal.

E.8532 Consider the parallelogram $ABCD$ shown below où the points I and J are the respective middles of the segments $[AB]$ and $[CD]$.



Using the points in the figure, give a representative of the sum: $2\cdot\overrightarrow{AJ}+2\cdot\overrightarrow{CB}$

E.8533 In the plane, consider A, B, C three non-aligned points of the plane.

For each question, determine the value of the real k verifying the equality:

(a) $3\cdot\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=k\cdot\overrightarrow{AB}$

(b) $3\overrightarrow{AB}-\overrightarrow{BC}+\overrightarrow{AC}+2\cdot\overrightarrow{BA}=k\cdot\overrightarrow{AB}$

5. *Vectors and distributivity*

E.2047

Proposition: in the plane, consider the two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ and a real k. We have the two identities below:
- $k\cdot\left(\overrightarrow{u}+\overrightarrow{v}\right)=k\cdot\overrightarrow{u}+k\cdot\overrightarrow{v}$
- $k\cdot\left(\overrightarrow{u}-\overrightarrow{v}\right)=k\cdot\overrightarrow{u}-k\cdot\overrightarrow{v}$

Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be two vectors. Simplify each of the following vector sums:

(a) $3\overrightarrow{u}-2\overrightarrow{v}+2\overrightarrow{u}-\overrightarrow{v}$ (b) $2\cdot\left(\overrightarrow{u}+\overrightarrow{v}\right)-\overrightarrow{u}$

(d) $-\left(\overrightarrow{u}+\overrightarrow{v}\right)+2\cdot\left(\overrightarrow{u}-\overrightarrow{v}\right)$ (e) $\dfrac{2}{3}\cdot\left(2\cdot\overrightarrow{u}-\dfrac{3}{2}\cdot\overrightarrow{v}\right)-\dfrac{1}{6}\overrightarrow{u}$

E.2947 Let A, B, C be three non-aligned points of the plane:

For each question, determine the value of the real k verifying the proposed relationship:

(a) $2\cdot\overrightarrow{AB}+2\overrightarrow{BC}+\overrightarrow{AC}=k\cdot\overrightarrow{AC}$ (b) $3\cdot\overrightarrow{AB}-3\cdot\overrightarrow{CB}=k\cdot\overrightarrow{AC}$

E.2048

(1) (a) Place three points A, B and C not aligned in the plane.

(b) Draw a representative of the sum:
$\overrightarrow{u}=-\overrightarrow{AB}-2\cdot\overrightarrow{BC}+2\cdot\overrightarrow{AC}$

(c) What conjecture can we make?

(2) Establish that: $\overrightarrow{u}=\overrightarrow{AB}$

Hint: We'll use the Chasles relationship:
$\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}$

E.9720 In the plane, consider A, B, C three non-aligned points of the plane.

For each question, determine the value of the real k verifying the equality:

(a) $2\cdot\overrightarrow{AB}+2\cdot\overrightarrow{BC}+\overrightarrow{AC}=k\cdot\overrightarrow{AC}$

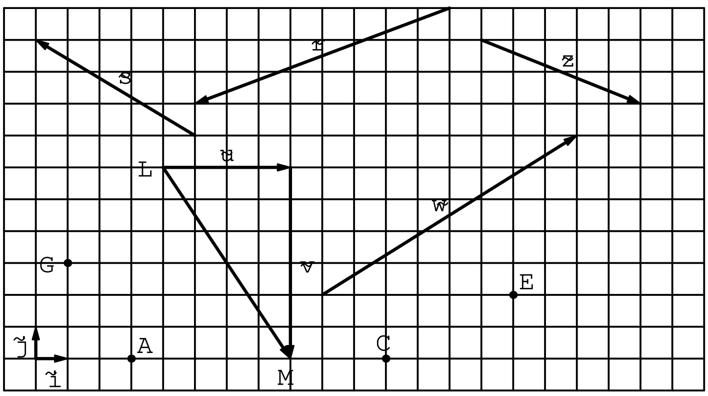
(b) $\overrightarrow{AB}+2\cdot\overrightarrow{AC}+4\overrightarrow{BC}=k\cdot\left(\overrightarrow{AC}+\overrightarrow{BC}\right)$

6. *Decomposition in a vector basis*

E.937

Definition: vectors $\overrightarrow{i}$ and $\overrightarrow{j}$ are said to form a vector basis if they have different directions.

The graph below shows two vectors $\overrightarrow{i}$ and $\overrightarrow{j}$ with different directions. The aim of this exercise is to decompose any vector in the plane as a function of the vectors $\overrightarrow{i}$ and $\overrightarrow{j}$.



(1) (a) Draw a representative of the vector $\overrightarrow{y}$ defined by:
$\overrightarrow{y}=4\times\overrightarrow{i}$

(b) Place the point B such that: $\overrightarrow{AB}=\overrightarrow{y}$

(2) (a) Place the point D such that: $\overrightarrow{CD}=-\overrightarrow{i}$.

(b) Place the point F such that: $\overrightarrow{EF}=-3\cdot\overrightarrow{j}$

(3) (a) Place the point H such that: $\overrightarrow{GH}=2\cdot\overrightarrow{i}$.

(b) Place the point K such that: $\overrightarrow{HK}=4\cdot\overrightarrow{j}$.

(c) Complete the following equality:
$\overrightarrow{GK}=\dots\dots\cdot\overrightarrow{i}+\dots\dots\cdot\overrightarrow{j}$

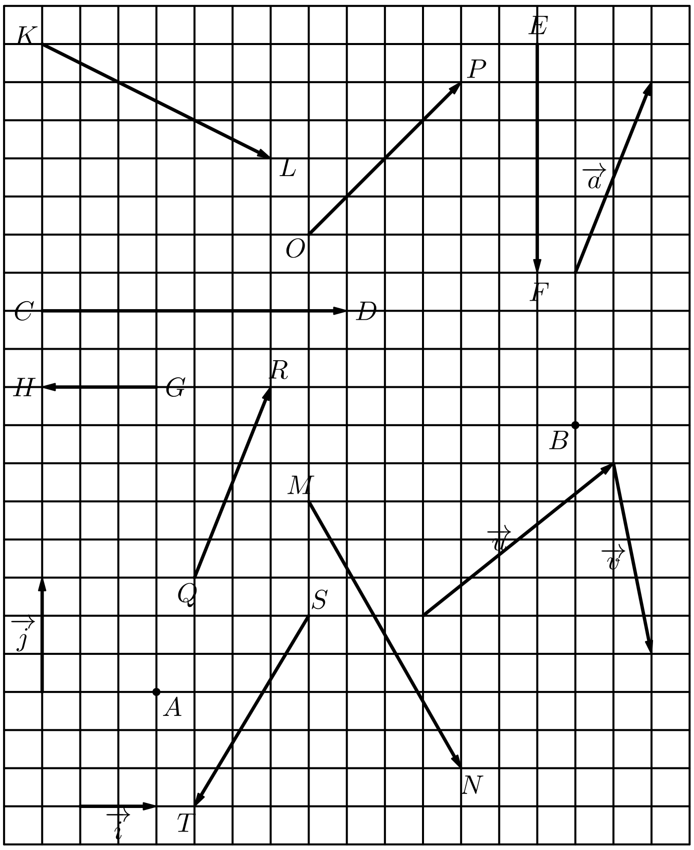
(4) Complete the following dotted lines:
$\overrightarrow{u}=\dots\dots\cdot\overrightarrow{i}$; $\overrightarrow{v}=\dots\dots\cdot\overrightarrow{j}$
$\overrightarrow{LM}=\dots\dots\cdot\overrightarrow{i}+\dots\dots\cdot\overrightarrow{j}$

(5) Complete the following dotted lines:

(a) $\overrightarrow{w}=\dots\cdot\overrightarrow{i}+\dots\cdot\overrightarrow{j}$ (b) $\overrightarrow{z}=\dots\cdot\overrightarrow{i}+\dots\cdot\overrightarrow{j}$

(c) $\overrightarrow{r}=\dots\cdot\overrightarrow{i}+\dots\cdot\overrightarrow{j}$ (d) $\overrightarrow{s}=\dots\cdot\overrightarrow{i}+\dots\cdot\overrightarrow{j}$

E.486



1. Place the point W such that: $\overrightarrow{AW}=-\vec{i}+2\cdot\vec{j}$.
   Place the point Z such that: $\overrightarrow{BZ}=-2\,\vec{i}+\dfrac{5}{3}\cdot\vec{j}$

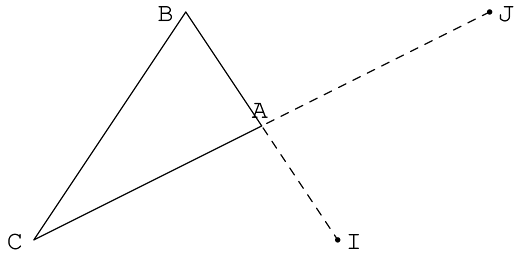
The vectors $\vec{i}$ and $\vec{j}$ are two vectors of different directions. For any vector $\vec{u}$ of the plane, there exist two real numbers α and β achieving equality:

$$\vec{u}=\alpha\cdot\vec{i}+\beta\cdot\vec{j}$$

This decomposition is called "*linear combination*".

2. a. Determine the linear combination of each of the vectors shown in the graph as a function of the vectors $\vec{i}$ and $\vec{j}$.
   b. Deduce the equality of the vectors $\overrightarrow{QR}$ and $\vec{a}$?
3. a. Draw a representative of the vector $\vec{u}+\vec{v}$.
   b. Graphically, express the vector $\vec{u}+\vec{v}$ using the vectors $\vec{i}$ and $\vec{j}$.
   c. Find this decomposition using those of the vectors $\vec{u}$ and $\vec{v}$ obtained in question 2.

E.5290 In the plane, consider any triangle ABC. Note respectively I and J the respective symmetries of B and C with respect to A:



In the reference frame $\left(A\,;\overrightarrow{AB}\,;\overrightarrow{AC}\right)$, give the coordinates of the following vectors:

a. $\overrightarrow{IA}$ b. $\overrightarrow{AJ}$ c. $\overrightarrow{BC}$

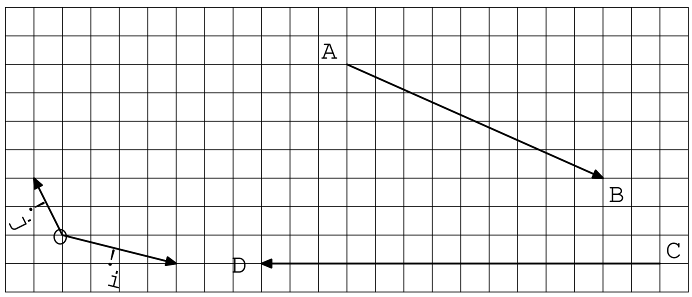
d. $\overrightarrow{CB}$ e. $\overrightarrow{IJ}$ f. $\overrightarrow{IC}$

E.9808 In the plane, consider the three points A, B, C. Consider the vector $\vec{v}$ defined by:

$$\vec{v}=\frac{5}{6}\cdot\overrightarrow{AC}-\frac{1}{2}\cdot\overrightarrow{AB}-\frac{1}{3}\cdot\overrightarrow{BC}+\frac{1}{3}\cdot\overrightarrow{CA}$$

1. Express the vector $\vec{v}$ as a function of $\overrightarrow{AB}$ and $\overrightarrow{AC}$:
2. In the reference frame $\left(A\,;\overrightarrow{AB}\,;\overrightarrow{AC}\right)$, give the coordinates of the vector $\vec{u}$

E.7215 In the plane, consider the reference frame $\left(O\,;\vec{i}\,;\vec{j}\right)$ and the two non-collinear vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$colinear vectors shown below:



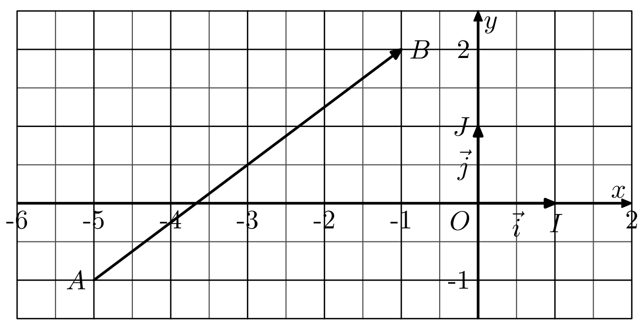
Without justification, give the decompositions of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ in the base $\left(\vec{i}\,;\vec{j}\right)$.

7. *Vector base and introduction to coordinate operations*

E.8534

Definition: in the plane equipped with a coordinate system $(O\,;I\,;J)$, we call the unit vector with abscissa *(or ordinate)*, noted $\vec{i}$ *(or $\vec{j}$)*, the vector $\overrightarrow{OI}$ *(resp. $\overrightarrow{OJ}$)*.

In the plane equipped with a coordinate system $(O\,;I\,;J)$, consider the two points A and B shown below:

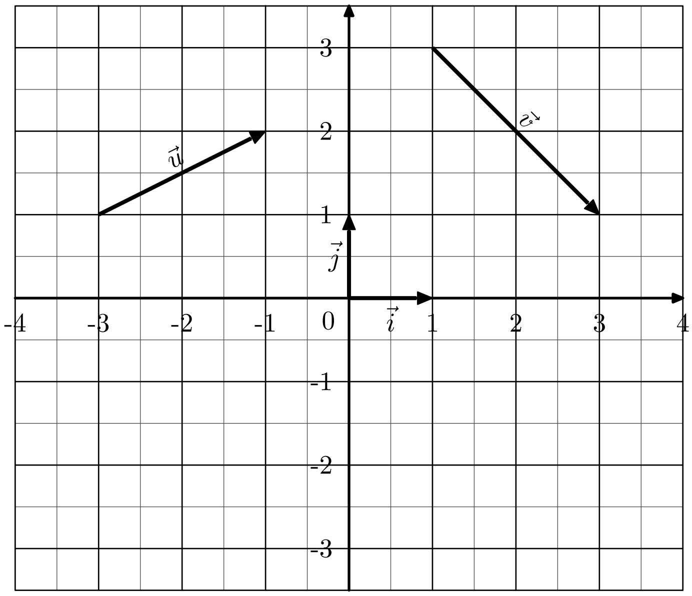


1. Decompose the vector $\overrightarrow{AB}$ into the vector basis $\left(\vec{i}\,;\vec{j}\right)$. That is, complete the dotted lines in the equation:

$\overrightarrow{AB} = \ldots\ldots \times \overrightarrow{i} + \ldots\ldots \times \overrightarrow{j}$

2. a. Give the coordinates of points A and B.
   b. Determine the coordinates of the vector $\overrightarrow{AB}$.
3. What comparison can be made between the coordinates of a vector and its decomposition in the vector basis of the unit vectors of the coordinate system?

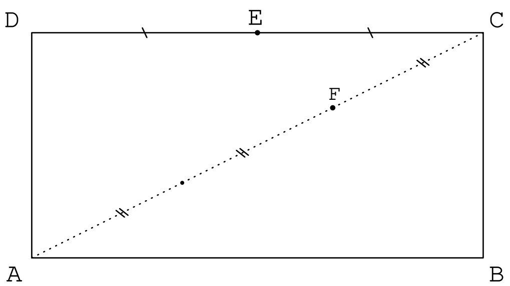
E.527 We equip the plane with an orthonormal coordinate system $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\right)$.



1. Give, without proof, the coordinates of the vectors $\overrightarrow{u}$ and $\overrightarrow{v}$.
2. a. Draw a line segment representing the vector $\overrightarrow{w}$ defined by:
   $\overrightarrow{w} = 3 \cdot \overrightarrow{u}$.
   b. Graphically, find the coordinates of $\overrightarrow{w}$.
   c. Compare the coordinates of vectors $\overrightarrow{u}$ and $\overrightarrow{w}$.
3. a. Draw a line segment representing vector $\overrightarrow{z}$ defined by:
   $\overrightarrow{z} = \overrightarrow{u} + \overrightarrow{v}$
   b. Graphically, find the coordinates of $\overrightarrow{z}$.
   c. Compare the coordinates of vector $\overrightarrow{z}$ relative to those of vectors $\overrightarrow{u}$ and $\overrightarrow{v}$.

8. *Coordinates of points*

E.9814 Consider the rectangle $ABCD$ shown below



Consider the plane provided with the reference frame $\left(A\,;\overrightarrow{AB}\,;\overrightarrow{AD}\right)$.

1. a. Complete the blanks in the equality:
   $\overrightarrow{AC} = \ldots\cdot\overrightarrow{AB} + \ldots\cdot\overrightarrow{AD}$
   Deduce the coordinates of point C.
   b. Give the coordinates of the points A, B, D.
2. The point E is the midpoint of the segment $[CD]$. Determine the coordinates of point E.
3. The F is defined by the equality: $\overrightarrow{CF} = \dfrac{1}{3}\cdot\overrightarrow{CA}$
   Give the coordinates of the point F.

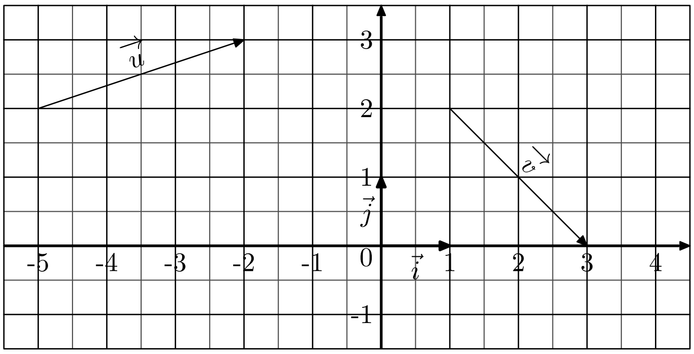
9. *Operations on coordinates*

E.8535

Proposition: Let $\overrightarrow{u}\,(x\,;y)$ and $\overrightarrow{v}\,(x'\,;y')$ be two vectors and k a real number $(k \in \mathbb{R})$.

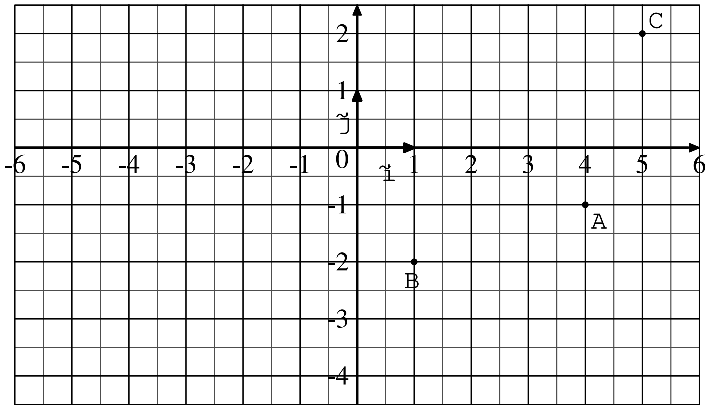
- The vector $-\overrightarrow{u}$, opposite to the vector $\overrightarrow{u}$, has the following coordinates: $-\overrightarrow{u} = (-x\,;-y)$
- The sum $\overrightarrow{u}+\overrightarrow{v}$ has the following coordinates:
  $\overrightarrow{u}+\overrightarrow{v}\,(x+x'\,;y+y')$
- The vector $k\times\overrightarrow{u}$ has the following coordinates:
  $k\times\overrightarrow{u}\,(k\times x\,;k\times y)$

In the plane with the coordinate system $\left(O\,;\,\vec{i}\,;\,\vec{j}\right)$, consider the two vectors $\vec{u}$ and $\vec{v}$ shown below:



1. Give the coordinates of the vectors $\vec{u}$ and $\vec{v}$.
2. Let $\vec{w}$ be the vector defined by: $\vec{w} = \vec{u} + 2\cdot\vec{v}$
   a. Give the coordinates of the vector $\vec{w}$.
   b. Draw the vector $\vec{w}$.
3. Let $\vec{z}$ be the vector defined by: $\vec{z} = \vec{u} - \vec{v}$
   a. Give the coordinates of the vector $\vec{z}$.
   b. Draw the vector $\vec{z}$

E.8536 We consider equipped with the reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\right)$ orthonormal and the three points A, B, C shown below:



1. a. Give, without justification, the coordinates of the vectors: $\overrightarrow{AB}$; $\overrightarrow{BC}$; $\overrightarrow{AC}$
   b. Determine the coordinates of the vector $\vec{u}$ defined by: $\vec{u} = 3\cdot\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA}$
2. Determine the unique real number k $(k\in\mathbb{R})$ verifying: $\vec{u} = k\times\overrightarrow{AB}$

E.8537 In the plane provided with a reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\right)$, consider the three points A, B, C defined by:

$A(2\,;-3)$; $B(-4\,;2)$; $C(0\,;-1)$

1. Determine the coordinates of the vector $\vec{u}$ defined by: $\vec{u} = 2\times\overrightarrow{AB} + 2\times\overrightarrow{BC} + \overrightarrow{AC}$
2. What simplified expression does the vector $\vec{u}$ admit?

10. *Operations and search for the coordinates of a point*

E.516 Consider a plane equipped with an arbitrary coordinate system $\left(O\,;\,\vec{i}\,;\,\vec{j}\right)$ and the following three points, determined by their coordinates: $A(2\,;1)$; $B(3\,;2)$

1. Determine the coordinates of the vector $3\cdot\overrightarrow{AB}$.
2. Find the coordinates of the point D such that:
   $\overrightarrow{AD} = 3\cdot\overrightarrow{AB}$.

E.307 In the plane with the origin $(O\,;I\,;J)$, consider the three points A, B, and C with coordinates:

$A(2\,;1)$; $B(-1\,;3)$; $C(0\,;-2)$

Determine the coordinates of point M that satisfy the following vector relationship: $\overrightarrow{CM} = 2\cdot\overrightarrow{AB}$

E.8539 Consider the plane provided with a $\left(O\,;\,\vec{i}\,;\,\vec{j}\right)$ any coordinate system and the following three points determined by their coordinates: $A(2\,;1)$; $B(3\,;2)$; $C(-1\,;-1)$

1. Determine the coordinates of the vector defined by the expression: $2\cdot\overrightarrow{AB} - 4\cdot\overrightarrow{AC}$
2. Determine the coordinates of the point E verifying the relation: $\overrightarrow{AE} = 2\cdot\overrightarrow{AB} - 4\cdot\overrightarrow{AC}$

E.8201 Consider the plane provided with a reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\right)$. Consider the three points A, B, C verifying the following relationships:

$2\,\overrightarrow{OA}(4\,;6)$; $3\,\overrightarrow{AB}(9\,;3)$; $2\,\overrightarrow{BC} - \overrightarrow{OB} = \vec{u}$

where the vector $\vec{u}$ has coordinates: $\vec{u}\,(3\,;3)$

Determine the coordinates of the points A, B and C.

E.8538 In the plane provided with a reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,\right)$, consider the three points A, B and C with coordinates:

$$A(2\,;1) \quad ; \quad B(-1\,;3) \quad ; \quad C(0\,;-2)$$

Determine the coordinates of the point N verifying the following vector relation: $4{\cdot}\overrightarrow{AN}-\overrightarrow{BN}-2{\cdot}\overrightarrow{CN}=\vec{0}$

E.518 Consider a plane equipped with an orthonormal coordinate system $(O\,;\,\vec{i}\,;\,\vec{j}\,)$ and the three points A, B, and C with coordinates $(-2\,;1)$, $(0\,;3)$, and $(3\,;0)$, respectively.

1. a. Find the coordinates of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
   b. Find the coordinates of the vector $\overrightarrow{AB}+\overrightarrow{AC}$.
2. Consider the point D that satisfies the relation: $\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{AD}$
   a. Let $(x_D\,;y_D)$ denote the coordinates of the point D. Justify that the following two equalities hold:
   $$\begin{cases} x_D+2=7 \\ y_D-1=1 \end{cases}$$
   b. Use this to find the coordinates of point D.

E.11715 Dans le plan muni d'un repère $(O\,;I\,;J)$, on considère les trois points A, B, C:

$$A(5\,;-2) \quad ; \quad B(-3\,;1) \quad ; \quad C(4\,;4)$$

Déterminer les coordonnées du point M tel que: $\overrightarrow{CM}=4{\cdot}\overrightarrow{AB}$

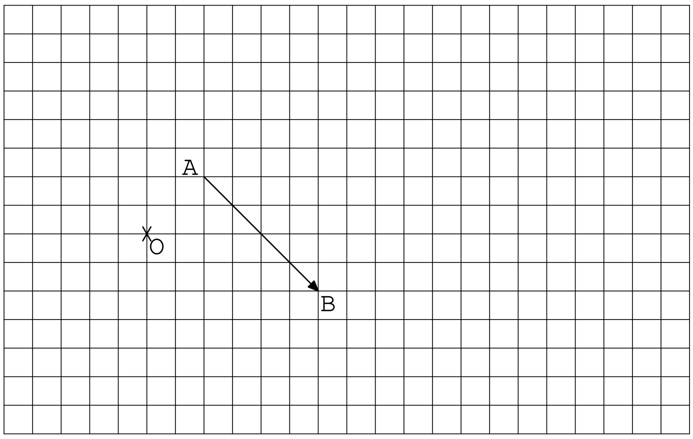
11. *Vector collinearity*

E.8543

> **Definition:** Let $\vec{u}$ and $\vec{v}$ be two non-zero vectors in the plane.
> Two vectors are said to be **collinear** if there exists a real number k such that: $\vec{u}=k{\cdot}\vec{v}$
>
> The real number k is called the **coefficient of collinearity** of $\vec{u}$ with respect to $\vec{v}$.

1. Let $\vec{u}$ and $\vec{v}$ be two vectors such that: $2\vec{u}=3\vec{v}$
   Justify that vectors $\vec{u}$ and $\vec{v}$ are collinear and that their collinearity coefficient is $\dfrac{3}{2}$.
2. Let $\vec{u}$ and $\vec{v}$ be two vectors such that: $\vec{u}+\vec{v}=\vec{0}$.
   Justify that these two vectors are collinear.
3. For each of the questions below, vectors $\vec{u}$ and $\vec{v}$ are collinear. Determine the value of the collinearity coefficient of $\vec{u}$ with respect to $\vec{v}$:
   a. $\dfrac{1}{2}{\cdot}\vec{u}=\dfrac{3}{4}{\cdot}\vec{v}$ b. $3{\cdot}\vec{u}-2{\cdot}\vec{v}=\vec{0}$

E.6998 In the plane, consider the three points O, A, B below and the vector $\overrightarrow{AB}$:



1. a. Draw the vector $\overrightarrow{A'B'}$ image of the vector $\overrightarrow{AB}$ by the homothety of center O and ratio 3.
   b. Draw the vector $\overrightarrow{A''B''}$ image of the vector $\overrightarrow{AB}$ by the homothety of center O and ratio $-\dfrac{1}{2}$.
2. What can we say about the vectors $\overrightarrow{AB}$, $\overrightarrow{A'B'}$ and $\overrightarrow{A''B''}$?

E.520 Consider the plane equipped with a coordinate system $\left(O\,;\,\vec{i}\,;\,\vec{j}\,\right)$. For each question, determine whether the two vectors $\vec{u}$ and $\vec{v}$ are collinear.
If they are, give the associated collinearity coefficient of $\vec{u}$ with respect to $\vec{v}$:

a. $\vec{u}\begin{pmatrix}-1\\2\end{pmatrix}$; $\vec{v}\begin{pmatrix}4\\-8\end{pmatrix}$ b. $\vec{u}\begin{pmatrix}3\\2\end{pmatrix}$; $\vec{v}\begin{pmatrix}9\\4\end{pmatrix}$

c. $\vec{u}\begin{pmatrix}2\\3\end{pmatrix}$; $\vec{v}\begin{pmatrix}4{,}2\\6{,}3\end{pmatrix}$ d. $\vec{u}\begin{pmatrix}0{,}7\\4{,}1\end{pmatrix}$; $\vec{v}\begin{pmatrix}-2{,}8\\16{,}4\end{pmatrix}$

E.5295 For each question, specify whether the vectors $\vec{u}$ and $\vec{v}$ are collinear and, if so, give the coefficient of collinearity of the vector $\vec{u}$ with respect to the vector $\vec{v}$:

a. $\vec{u}(-2\,;-10)$ et $\vec{v}(4\,;20)$ b. $\vec{u}(0\,;5)$ et $\vec{v}(-5\,;0)$

E.8202 In the plane provided with a reference frame $(O\,;\vec{i}\,;\vec{j})$, consider the five points:
$A(2\,;-2)$; $B(11\,;-14)$; $C(-3\,;1)$; $D(5\,;3)$; $E(12\,;-19)$

Of the four vectors above, only one is collinear with the vector $\overrightarrow{AB}$. Which is it? Justify your answer.

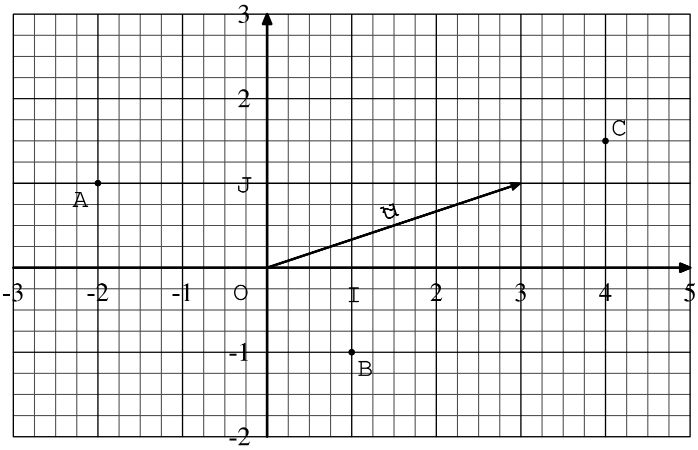
$$\overrightarrow{BC}\ ;\ \overrightarrow{CD}\ ;\ \overrightarrow{DE}\ ;\ \overrightarrow{CE}$$

E.9813 For each question, specify whether the vectors $\vec{u}$ and $\vec{v}$ are collinear and, if so, give the coefficient of collinearity of the vector $\vec{u}$ with respect to the vector $\vec{v}$:

a) $\vec{u}\,(-6\,;9)$ et $\vec{v}\left(\dfrac{1}{4}\,;-\dfrac{1}{2}\right)$ b) $\vec{u}\left(-\dfrac{4}{3}\,;4\right)$ et $\vec{v}\,(3\,;-9)$

c) $\vec{u}\left(\dfrac{1}{3}\,;\dfrac{2}{5}\right)$ et $\vec{v}\,(5\,;6)$ d) $\vec{u}\,(6\,;-5)$ et $\vec{v}\left(\dfrac{14}{5}\,;-2\right)$

E.6624 The plane is given a reference frame $(O\,;I\,;J)$ and the points A, B and C below are considered:



1) a) Give the coordinates of points A, B and C.

b) Determine the coordinates of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

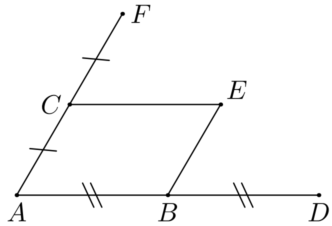
c) Deduce the coordinates of the vector $\vec{v}$ defined by:
$\vec{v}=\overrightarrow{AB}+2\cdot\overrightarrow{BC}$

2) Justify that the vectors $\vec{u}$ and $\vec{v}$ are collinear.

E.500

In the plane, consider the points A, B, C, D and E such that:

- C is the middle of $[AF]$; B is the middle of $[AD]$.
- The quadrilateral $ABEC$ is a parallelogram.



Let's equip the plane with the reference frame $(A\,;B\,;C)$ any.

1) In the reference frame $(A\,;B\,;C)$, give, without justification, the coordinates of the six points in this plane.

2) Justify that the points E, F and D are aligned.

12. *Colinearity and algebraic manipulation*

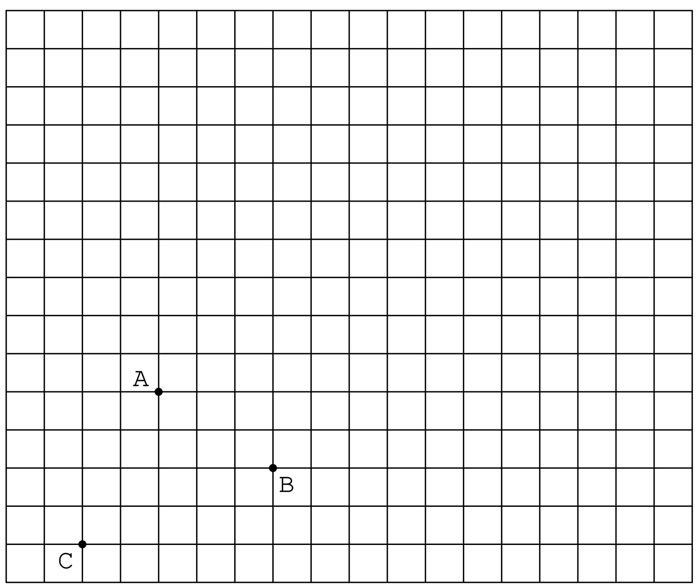
E.9718 Let A, B, C and D be four points of the plane verifying the relation:
$\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$

Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

E.9812 For each of the questions below, the vectors $\vec{u}$ and $\vec{v}$ are collinear. Determine the value of the collinearity coefficient of $\vec{u}$ with respect to $\vec{v}$:

a) $3\cdot(\vec{u}-2\cdot\vec{v})=\vec{0}$ b) $-2\cdot(\vec{u}+\vec{v})=2\cdot\vec{u}+3\cdot\vec{v}$

E.4812 Consider the three points A, B and C shown in the grid below:



1) a) Place the point M verifying the vector relation:
$\overrightarrow{AM}=2\cdot\overrightarrow{CA}$

(b) Place the point N verifying the vector relation:
$\overrightarrow{AN} = \overrightarrow{AB} + 2\cdot\overrightarrow{CB}$

(c) What conjecture can be made about the vectors $\overrightarrow{AB}$ and $\overrightarrow{MN}$?

(2) Demonstrate, using vector calculus, establish the equality:
$\overrightarrow{MN} = 3\cdot\overrightarrow{AB}$

E.9717 Let A, B, C and D be four points of the plane such that:
$\overrightarrow{AD} + \overrightarrow{BD} + 2\cdot\overrightarrow{CB} = \overrightarrow{0}$

(1) Establish equality: $\overrightarrow{AB} = -2\cdot\overrightarrow{CD}$

(2) What can be said about the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$?

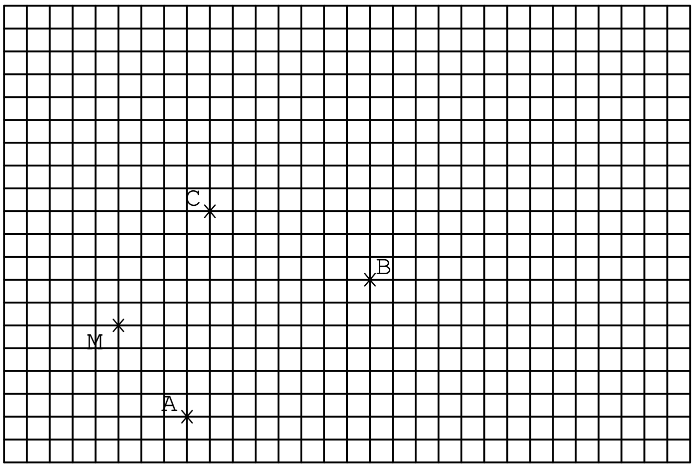
E.510 Let A, B, C and D be four points of the plane verifying the relation:
$\overrightarrow{AC} - 3\cdot\overrightarrow{BD} + 2\cdot\overrightarrow{BC} = \overrightarrow{0}$

Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

E.501 In each case, consider three points A, B, C of the plane verifying a vector relationship. Show that in each case, the points A, B and C are aligned:

(a) $3\cdot\overrightarrow{AB} + \overrightarrow{BC} = -2\cdot\overrightarrow{AC}$ (b) $-2\cdot\overrightarrow{AB} = 3\times\left(\overrightarrow{CB} + \overrightarrow{CA}\right)$

E.2917 In the plane, shown below fitted with a grid, consider the points A, B, C, M:



Give a representative of the vector $\overrightarrow{u}$ defined by the relation:
$\overrightarrow{u} = 2\cdot\overrightarrow{AB} + \overrightarrow{CB} - \overrightarrow{AC}$

(1) Place the point N such that: $\overrightarrow{MN} = \overrightarrow{u}$.

(2) We define the vector $\overrightarrow{v}$ defined by: $\overrightarrow{v} = \overrightarrow{CB} + \frac{1}{3}\cdot\overrightarrow{AC}$

Show that the vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ are collinear.

E.5293 Let A, B, C and D be four points in the plane such that:
$5\cdot\overrightarrow{AD} = 2\cdot\overrightarrow{AC} + 3\cdot\overrightarrow{BD}$

Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

E.9719 Let A, B, C and D be four points in the plane such that:
$3\cdot\overrightarrow{AD} + 4\cdot\overrightarrow{BC} = 7\cdot\overrightarrow{AC}$

Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

E.8122 Consider the four points A, B, C and D verifying the vector relation:
$2\cdot\overrightarrow{DC} + 5\cdot\overrightarrow{CB} + 5\cdot\overrightarrow{AD} - 3\cdot\overrightarrow{AB} = \overrightarrow{0}$

Demonstrate that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

E.2055 Let A, B, C be three points in the plane verifying the relation:
$-\frac{1}{2}\cdot\overrightarrow{AB} + \frac{5}{2}\cdot\overrightarrow{BC} - \overrightarrow{BA} + \overrightarrow{CB} = \overrightarrow{0}$

(1) Show that these three points verify: $\overrightarrow{AB} = \frac{3}{2}\cdot\overrightarrow{AC}$

(2) What can be said about the points A, B, C?

E.5343 In the plane, consider a ABC non-aplatized triangle. Consider the three points M, N and P defined by:
$\overrightarrow{BM} = \frac{1}{3}\cdot\overrightarrow{BA}$; $\overrightarrow{BN} = \frac{1}{2}\cdot\overrightarrow{BC}$; $\overrightarrow{AP} = 2\cdot\overrightarrow{AC}$

Show that the points M, N and P are aligned.

E.2903 Let A, B, C be three points in the plane. Show that the vector $\overrightarrow{u}$ defined below is collinear with the vector $\overrightarrow{AC}$ by:
$\overrightarrow{u} = 3\cdot\overrightarrow{AB} + \frac{2}{3}\cdot\overrightarrow{BC} - \frac{5}{3}\cdot\overrightarrow{CA} + \frac{7}{3}\cdot\overrightarrow{BA}$

E.5294 ⚠ Consider a triangle ABC and M a point belonging to side $[AB]$ verifying the relation: $AM = \frac{2}{3}\cdot AB$

P is the point of intersection of the line (BC) and the parallel to (AC) passing through the point M. N is the point of intersection of the straight lines (AC) and the parallel to (AB) passing through the point P

(1) Make a representation of this configuration.

(2) Show that: $AN = \frac{1}{3}\cdot AC$; $CP = \frac{2}{3}\cdot CB$.

(3) Decompose the vectors below in terms of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$:

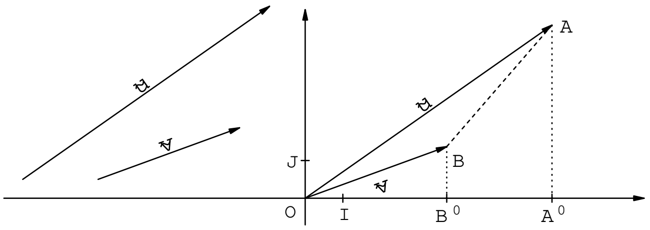
(a) $\overrightarrow{AP}$ (b) $\overrightarrow{MC}$

(4) Decompose the vectors below in terms of the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$:

(a) $\overrightarrow{AP}$ (b) $\overrightarrow{NM}$

13. Introduction to the collinearity criterion

E.8364 In the plane provided with a reference frame $(O\,;I\,;J)$, consider two vectors $\overrightarrow{u}(x\,;y)$ and $\overrightarrow{v}(x'\,;y')$ such that:
$0 < x' < x$; $0 < y' < y$



Consider the two points A and B such that:
$\overrightarrow{OA} = \vec{u}$; $\overrightarrow{OB} = \vec{v}$

1. a. Express the areas of the following figures in terms of x, x', y and y':
OBB' ; OAA' ; $AA'B'B$
   b. Deduce the expression for the area of triangle OAB as a function of x, x', y and y'.
2. a. What can be said about the points O, A, B when: $x \times y' - x' \times y = 0$?
   b. Is the reciprocal true?

14. *Criteria of collinearities*

E.8541

> **Definition:** Let $\vec{u}(x\,;y)$ and $\vec{v}(x'\,;y')$ be vectors. We call **the determinant of vectors $\vec{u}$ and $\vec{v}$**, noted $\det(\vec{u}\,;\vec{v})$, defined by:
> $\det(\vec{u}\,;\vec{v}) = x \times y' - x' \times y$

For each of the pairs of vectors $\vec{u}$ and $\vec{v}$ defined below, determine the value of $\det(\vec{u}\,;\vec{v})$:

a. $\vec{u}(2\,;-1)$; $\vec{v}(3\,;4)$ b. $\vec{u}(-5\,;1)$; $\vec{v}(2\,;-2)$

E.11689 Dans le plan muni d'un repère $(O\,;\vec{i}\,;\vec{j})$, on considère les points: $A(-3\,;2)$; $B(1\,;4)$; $C(5\,;-1)$

Déterminer la valeur de $\det\big(\overrightarrow{AB}\,;\overrightarrow{AC}\big)$.

E.5288

> **Proposition:** In the plane provided with a reference frame, consider the two vectors $\vec{u}$ and $\vec{v}$.
> The two vectors $\vec{u}$ and $\vec{v}$ are collinear with each other if, and only if, their determinant is zero.

Consider the plane provided with a reference frame $(O\,;\vec{i}\,;\vec{j})$ and the four points:

$A(3\,;-5)$; $B(1\,;-1)$; $C(13\,;2)$; $D(18\,;-8)$

Establish that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

E.504 Consider the plane equipped with a coordinate system $(O\,;\vec{i}\,;\vec{j})$ and the two vectors:

$$\vec{u}\begin{pmatrix}1-2\sqrt{3}\\ \sqrt{3}+\sqrt{2}\end{pmatrix} \quad ; \quad \vec{v}\begin{pmatrix}6-\sqrt{3}\\ -3-\sqrt{6}\end{pmatrix}$$

Prove that the vectors $\vec{u}$ and $\vec{v}$ are collinear.

E.512 In the plane marked with the reference point $(O\,;\vec{i}\,;\vec{j})$, consider the following two vectors:

$$\vec{u}\begin{pmatrix}\left(\sqrt{2}+\sqrt{3}\right)^2\\ 1-\sqrt{10}\end{pmatrix} \quad ; \quad \vec{v}\begin{pmatrix}5\sqrt{2}+4\sqrt{3}\\ \sqrt{2}-2\sqrt{5}\end{pmatrix}$$

Show that the vectors $\vec{u}$ and $\vec{v}$ are collinear.

E.8542 In the plane provided with a reference frame $(O\,;\vec{i}\,;\vec{j})$, consider the two collinear vectors:

$$\vec{u}\left(x+y\sqrt{2}\,;4\right) \quad ; \quad \vec{v}\left(2\sqrt{2}-1\,;-2\right)$$

where x and y are two relative integers.

Determine the values of x and y.

E.7888 The plane is given a reference frame $(O\,;\vec{i}\,;\vec{j})$ orthonormal and we consider the points:

$$A\left(\frac{1}{4}\,;\frac{1}{3}\right) \quad ; \quad B\left(1\,;\frac{5}{6}\right) \quad ; \quad C\left(-\frac{1}{2}\,;\frac{7}{6}\right)$$

Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are not two collinear vectors.

E.11716 Dans le plan muni d'un repère $(O\,;I\,;J)$, on considère les trois points:
$A(5\,;1)$; $B(9\,;3)$; $C(-9\,;-5)$

1. Déterminer les coordonnées des vecteurs $\overrightarrow{AB}$ et $\overrightarrow{AC}$.
2. Est ce que les vecteurs $\overrightarrow{AB}$ et $\overrightarrow{AC}$ sont colinéaires? Justifier votre réponse.

15. *Parallelism and collinearity*

E.499 In the plane, consider the coordinate system $(O\,;\vec{i}\,;\vec{j})$ and the points:

$O(49\,;-100)$; $P(14\,;5)$; $Q(1\,;-85)$; $R(-58\,;92)$

Determine whether the lines (OP) and (QR) are parallel.

E.5289 In the plane provided with a reference frame $(O\,;\vec{i}\,;\vec{j})$, consider the three points: $A(-3\,;-1)$; $B(1\,;5)$; $C(-1\,;2)$

Show that the points A, B, C are aligned.

E.6507 In the plane provided with a reference frame $(O\,;\,\vec{i}\,;\,\vec{j})$, consider the four points:

$$A(2\,;-5)\quad;\quad B(-2\,;2)\quad;\quad C(-4\,;5)\quad;\quad D\Big(2\,;-\frac{11}{2}\Big)$$

Justify that the straight lines (AB) and (CD) are parallel.

E.5296 In a plane with an orthonormal coordinate system $(O\,;\,\vec{i}\,;\,\vec{j})$.

Consider the following four points in the plane:

$A\big(-\sqrt{2}\,;-\sqrt{6}\big)$; $B\big(2\sqrt{2}\,;0\big)$; $C(-2\sqrt{3}\,;3)$; $D(0\,;5)$

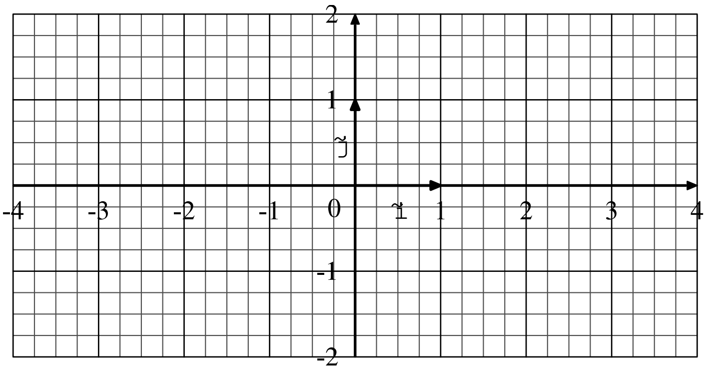
Show that the straight lines (AB) and (CD) are parallel.

E.8540 The plane is given a reference frame $(O\,;\,\vec{i}\,;\,\vec{j})$ and the three points:

$A(2\sqrt{2}+1;3+2\sqrt{2})$; $B(\sqrt{2}-1;\sqrt{2}+1)$; $C(\sqrt{2}+5;\sqrt{2}+7)$

Show that the points A, B and C are aligned.

E.5313 Consider the plane provided with the reference frame $(O\,;\,\vec{i}\,;\,\vec{j})$ shown below:



Consider the four vectors below:

$$\vec{u}\Big(\frac{9}{4}\,;-\frac{3}{4}\Big)\quad;\quad\vec{v}\Big(\frac{7}{2}\,;-\frac{3}{2}\Big)\quad;\quad\vec{w}\Big(-\frac{15}{4}\,;\frac{5}{4}\Big)$$

1. Represent the three vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ with the point O as their origin.
2. a. Conjecture the collinearity of vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ with each other.
   b. Establish your conjecture.

E.1144 In a plane with an orthonormal coordinate system $(O\,;\,\vec{i}\,;\,\vec{j})$, consider the points:

$$D(5\,;-2)\ ;\ E(-3\,;10)\ ;\ F(-3\,;-2)\ ;\ G(3\,;-11)$$

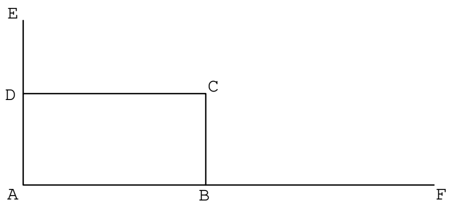
Show that the lines (DE) and (FG) are parallel.

Hint: To show that the lines (DE) and (FG) are parallel, it suffices to show that the vectors $\overrightarrow{DE}$ and $\overrightarrow{FG}$ are collinear.

16. *Modeling and collinearity*

E.9806 In the plane, consider the rectangle $ABCD$ such that $AB=4\,cm$ and $AD=2\,cm$ and the two points E and F verifying:

- $F\in[AB)$ and $AF=9\,cm$
- $E\in[AD)$ and $AE=3.6\,cm$

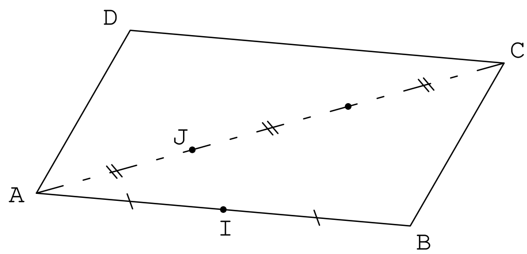


The plane is given the reference $\big(A\,;\overrightarrow{AB}\,;\overrightarrow{AD}\big)$.

1. a. Complete the following equalities:
   $\overrightarrow{AE}=\dots\overrightarrow{AD}$; $\overrightarrow{AF}=\dots\overrightarrow{AB}$
   b. In the reference frame $\big(A\,;\overrightarrow{AB}\,;\overrightarrow{AD}\big)$, give the coordinates of the six points of this plane.
2. Determine the coordinates of the vectors $\overrightarrow{CE}$ and $\overrightarrow{CF}$.
3. Deduce that the points C, E, F are aligned.

E.5824 In the plane, consider the parallelogram $ABCD$. Let I be the midpoint of segment $[AB]$ and J the point on segment $[AC]$ verifying the relation:

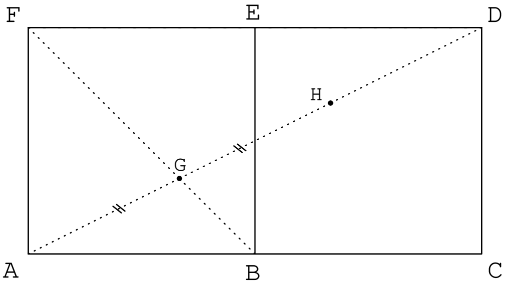
$$AJ=\frac{1}{3}\cdot AC$$



The plane is given the reference frame $\big(A\,;\,\overrightarrow{AB}\,;\,\overrightarrow{AD}\big)$.

1. Determine the coordinates of points D, I and J.
2. Demonstrate that the points D, I and J are aligned.

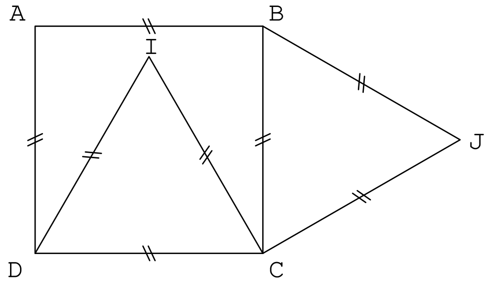
E.7214 Consider the figure below consisting of the two squares $ABEF$ and $BCDE$:



Note G the point of intersection of the straight lines (AD) and (BF) and H the point in the plane verifying the vector relation $\overrightarrow{AG}=\overrightarrow{GH}$

1. Placing yourself in the reference frame $\left(A\,;\overrightarrow{AB}\,;\overrightarrow{AF}\right)$, determine the coordinates of the point G.
2. Establish that the points E, H and C are aligned.

E.5394 Consider the figure above composed of a square $ABCD$ and two equilateral triangles DIC and BJC:



In this question any trace of research, however incomplete, or initiative however unsuccessful, will be taken into account in the assessment.

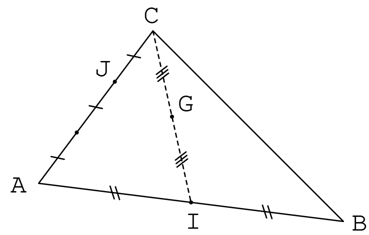
Show that the points A, I, J are aligned.

(In an equilateral triangle of side a, we admit that all its heights have length $\dfrac{a\sqrt{3}}{2}$).

E.5393

Consider the triangle opposite où I and G are the respective middles of the segments $[AB]$ and $[CI]$, the point J is defined by the relation:

$$\overrightarrow{CJ}=\frac{1}{3}\cdot\overrightarrow{CA}$$



Consider the vector basis $\left(\overrightarrow{AB}\,;\,\overrightarrow{AC}\right)$.

1. Express the vectors $\overrightarrow{AI}$ and $\overrightarrow{AJ}$ in the vector basis $\left(\overrightarrow{AB}\,;\,\overrightarrow{AC}\right)$.
2. Establish that the vector decomposition of the vector $\overrightarrow{AG}$:
$$\overrightarrow{AG}=\frac{1}{4}\cdot\overrightarrow{AB}+\frac{1}{2}\cdot\overrightarrow{AC}$$
3. Deduce the alignment of points B, G, J.

17. *Colinearity and coordinate search*

E.5291 Consider the plane provided with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\right)$.

Let A, B, C and D be four points in the plane with coordinates:

$$A(-5\,;1)\quad;\quad B(2\,;4)\quad;\quad C(-1\,;-2)\quad;\quad D(3\,;y_D)$$

Determine the coordinates of the point D such that the straight lines (AB) and (CD) are parallel and the point D has 3 as its abscissa.

E.8200 Consider the plane provided with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\right)$.

Let A, B and C be three points in the plane with coordinates: $(4\,;-1)\quad;\quad(1\,;3)\quad;\quad(1\,;-2)$

Determine the coordinates of the point D such that the straight lines (AB) and (CD) are parallel and the point D has 3 as its abscissa.

E.5314 In a plane with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\right)$, consider the three points A, B, C with coordinates:

$$A(1\,;2)\quad;\quad B\left(-2\,;\frac{5}{2}\right)\quad;\quad C(-1\,;4)$$

Determine the value of x so that the point D of coordinates $(x\,;3)$ is such that the straight lines $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

E.5822 In the plane provided with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\right)$ orthonormal, consider the following three points:

$$A(-1\,;1)\quad;\quad B(-3\,;-1)\quad;\quad C(2\,;3)$$

1. Are the points A, B and C aligned? Justify your answer.
2. Determine the coordinates of the single point D with abscissa -2 such that the straight lines (AB) and (CD) are parallel.

E.5746 Consider the plane with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\right)$ and the four points:

$$A(-3\,;2)\quad;\quad B(2\,;-1)\quad;\quad C(1\,;5)\quad;\quad D(7\,;2)$$

1. Are the straight lines (AB) and (CD) parallel?
2. Determine the coordinates of the point E with abscissa 7 so that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CE}$ are collinear.

E.2080 Consider the plane with any reference point $(O;\vec{i};\vec{j})$ and the following three points in the plane:

$A(3;2)\quad;\quad B(-1;3)\quad;\quad D(-4;-1)$

1. Determine the coordinates of the point C such that $ABCD$ is a parallelogram.
2. Determine the coordinates of the point E belonging to the x-axis such that: $(BD)/\!/(AE)$.

E.507 Consider a plane with a coordinate system $(O;\vec{i};\vec{j})$ and the three points:

$A\left(4+4\sqrt{2};3+3\sqrt{2}\right);B\left(\sqrt{2}-2;2\sqrt{2}+1\right);C\left(\sqrt{2}+1;y_C\right)$

where y_C is a real number.

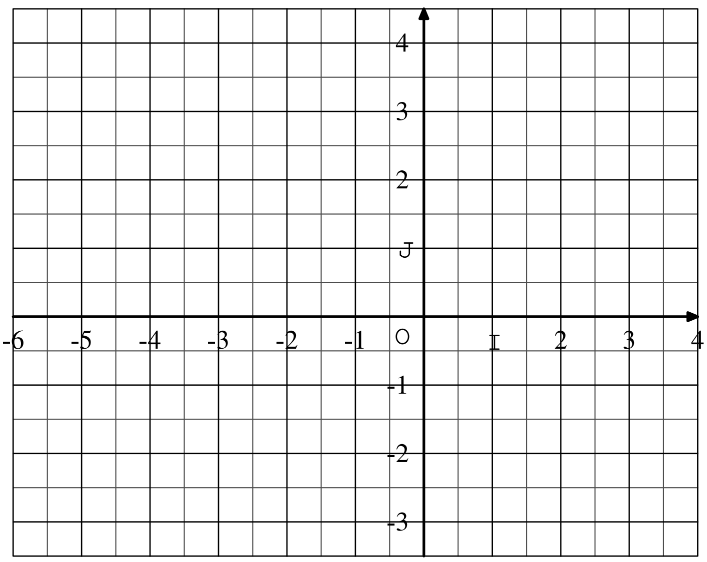
Given that the points A, B, and C lie on a straight line, determine the y-coordinate of point C in the form $a+b\sqrt{2}$, where a and b are two real numbers.

E.6625 Consider the plane provided with a reference frame $(O;\vec{i};\vec{j})$.
Let A, B, C, D be four points in the plane with respective coordinates: $A\left(2;\frac{8}{3}\right)$; $B\left(\frac{2}{3};2\right)$; $C\left(\frac{4}{5};0\right)$; $D\left(x_D;-\frac{1}{2}\right)$
where x_D is a real number.

Knowing that the straight lines (AB) and (CD) are parallel, determine the coordinates of the point D.

18. Vector geometry

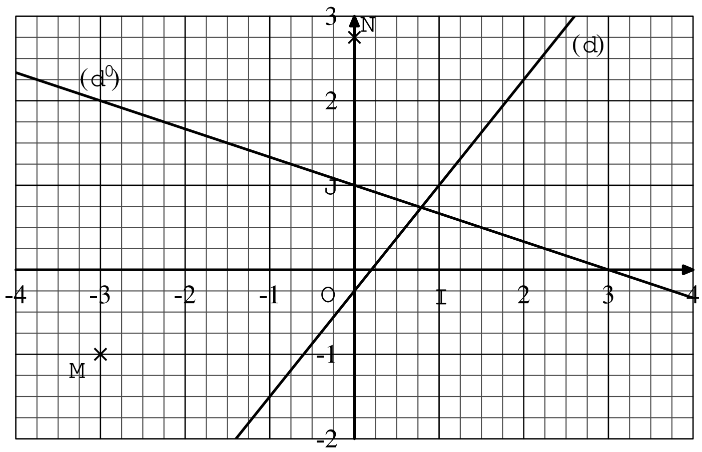
E.2107 Consider the plane provided with a $(O;\vec{i};\vec{j})$ orthonormal datum:



1. Place the three points A, B, C in the frame below:

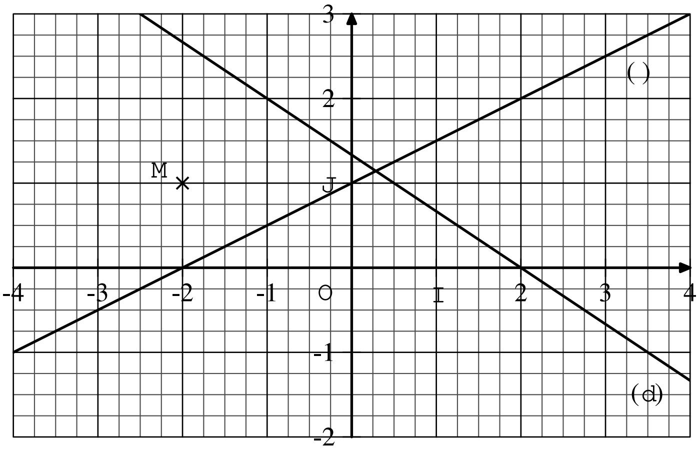
   $A(3;-3)\quad;\quad B(-4;3)\quad;\quad C(-5;-1)$
2. Determine the coordinates of the middle M of segment $[AB]$.
3. a. Determine lengths AB and MC
   b. Establish that the triangle ABC is right-angled at C.
4. Note N the point of intersection of the ordinate axis with the line parallel to (CM) passing through the point B.
   a. Place the point N in the marker.
   b. Determine the coordinates of point N.

E.2918 In the plane provided with the reference frame $(O;I;J)$, consider the straight lines (d) and (d') below:



1. Graphically determine the slope-intercept forms of the straight lines (d) and (d').
2. a. Give graphically the coordinates of the points M and N.
   b. Justify that the line (MN) is parallel to the line (d).
3. Let Q be a point on the line (d) such that the line (MQ) is parallel to (d'). We note x the abscissa of point Q.
   a. Justify that the vectors $\overrightarrow{MQ}$ and $\vec{u}\left(1;-\frac{1}{3}\right)$ are collinear.
   b. Justify that the vector $\overrightarrow{MQ}$ has coordinate as a function of x: $\overrightarrow{MQ}\left(x+3;\frac{5}{4}\cdot x+\frac{3}{4}\right)$
   c. Solve the equation: $x+3=-3\cdot\left(\frac{5}{4}\cdot x+\frac{3}{4}\right)$.
   d. Deduce the coordinates of point Q.

E.2902 In the plane provided with a $(O\,;I\,;J)$ orthonormal, consider the line (d) shown below and the point M with coordinate $(-2\,;1)$:



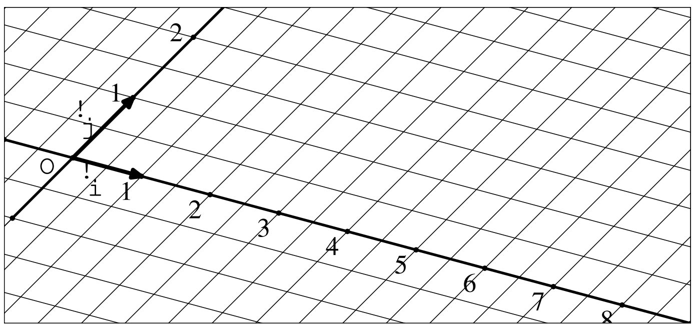
1. Determine the reduced equation of the line (d).
2. a. Draw a representative of the vector $\overrightarrow{u}\,(2\,;1)$.
   b. Determine the coordinates of the point P belonging to the line (d) such that the vectors $\overrightarrow{MP}$ and $\overrightarrow{u}$ are collinear.
3. Solve the following equation:
   $$(1-2x)\ (10x+13) = (x+2)\ (15x+24)$$
4. Let N be the point with coordinates $\left(-\frac{13}{10}\,;\frac{1}{5}\right)$. Let x be a real number, consider the two points R and S belonging respectively to the lines (d) and (Δ) each having as abscissa the value x
   a. We admit that the reduced equation of the line (Δ) is:
   $y=\frac{1}{2}\cdot x+1$
   Express as a function of x the coordinates of the two vectors $\overrightarrow{MR}$ and $\overrightarrow{NS}$

   We wish to determine a value of x for which the vectors $\overrightarrow{MR}$ and $\overrightarrow{NS}$ are collinear.

   Let us now assume that x verifies this constraint:
   b. Justify that x verifies the following condition:
   $$\left(x+\frac{13}{10}\right)\left(-\frac{2}{3}x+\frac{1}{3}\right)=\left(x+2\right)\left(\frac{1}{2}x+\frac{4}{5}\right)$$
   c. Deduce the coordinates of points R and S.

19. *Deepening: any benchmarks*

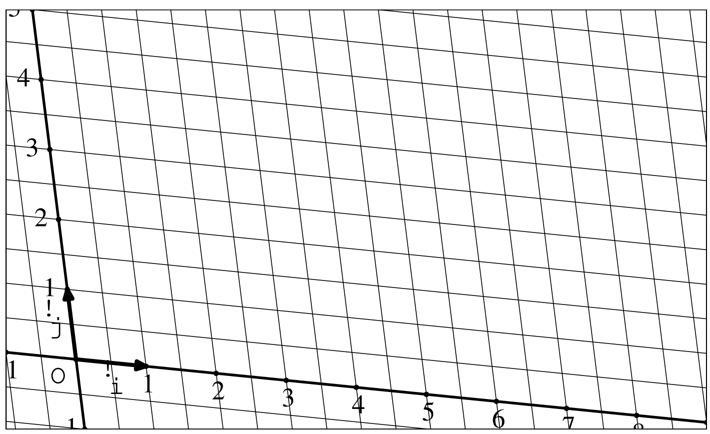
E.4968 The plane is provided with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\right)$ any represented below:



1. a. In the marker below, place the two points:
   $A(-1\,;2)\quad;\quad B(4\,;1)$
   b. Justify graphically that the vector $\overrightarrow{AB}$ has coordinates $(5\,;-1)$.
2. Consider the following two vectors:
   $\overrightarrow{u}\,(3\,;2)\quad;\quad\overrightarrow{v}\,(-2\,;-2)$

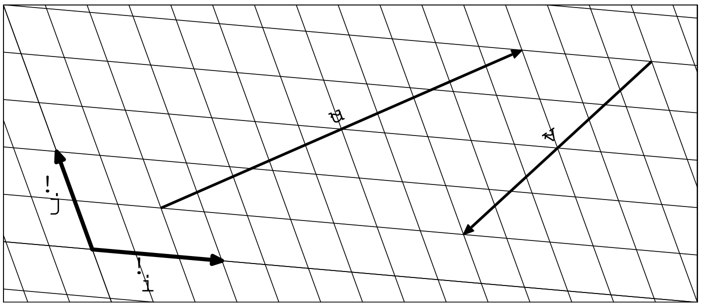
   Give a representative of your choice of each of these two vectors in the above reference frame.

E.5339 The plane is provided with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\right)$ any represented below:



1. Draw a representative of each of the two vectors:
   $\overrightarrow{u}\,(5\,;2)\quad;\quad\overrightarrow{v}\,(-3\,;-2)$
2. a. Draw a representative of the vector $\overrightarrow{w}$ defined by:
   $\overrightarrow{w}=\overrightarrow{u}+\overrightarrow{v}$
   b. Graphically, determine the coordinates of the vector $\overrightarrow{w}$.
   c. Compare the coordinates of the vector $\overrightarrow{w}$ relative to those of the vectors $\overrightarrow{u}$ and $\overrightarrow{v}$.

E.5744 In the plane, consider the two non-colinear vectors $\vec{i}$ and $\vec{j}$ shown below:

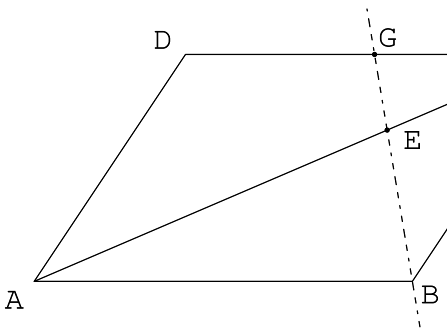


The vectors $\vec{u}$ and $\vec{v}$ are also shown above.

1. In the vector basis of $(\vec{i}\,;\vec{j})$, give the coordinates of the vectors $\vec{u}$ and $\vec{v}$.
2. By the method of your choice, determine the coordinates of the sum vector: $\vec{w}=\vec{u}+\vec{v}$.
3. By the method of your choice, determine the coordinates of the vector $\vec{t}$ achieving the following equality:
   $\vec{v}=\vec{u}+\vec{t}$

20. *Deepening: any base and affine function*

E.4592 Let $ABCD$ be a parallelogram. Let E be the point belonging to segment $[AC]$ verifying:
$AE=\frac{2}{3}\cdot AC$
The straight lines (BE) and (CD) intersect at the point G.



The plane is provided with the reference frame $\big(A\,;\overrightarrow{AB}\,;\overrightarrow{AC}\big)$.

1. Give, without justification, the coordinates of the points:
   A ; B ; C ; D ; E
2. a. Justify that the straight line (BE) has the equation:
   $y=-\frac{2}{3}\cdot x+\frac{2}{3}$
   b. Deduce the coordinates of point G.
3. What does the point G represent for the segment $[CD]$? Justify.

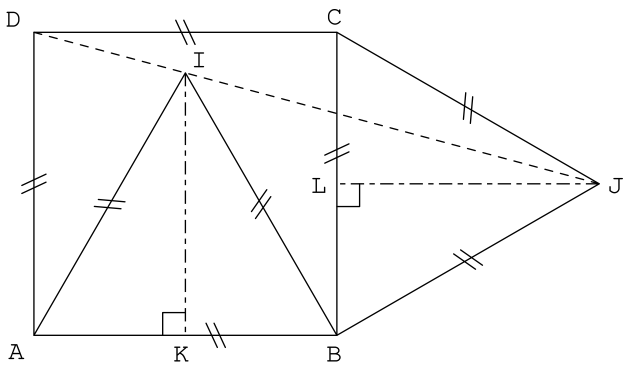
E.2074 Consider a square $ABCD$ of side $1\,cm$.

- The point I is the only point such that the triangle AIB is equilateral and is contained in the square $ABCD$;
- The point J is the only point such that the triangle BCJ is equilateral and is outside the square $ABCD$.

We note:

- K is the foot of the height from the vertex I in the triangle AIB.
- L is the foot of the height from J in the triangle BCJ.

Here is the representation of this configuration:



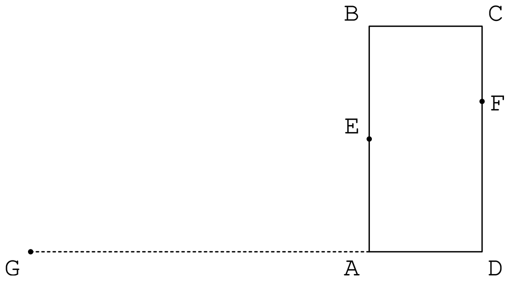
The purpose of the exercise is to show that the points D, I, and J are aligned.

Consider the plane munite of the reference frame $(A\,;\overrightarrow{AB}\,;\overrightarrow{AD})$.

1. What is the nature of the benchmark $(A\,;\overrightarrow{AB}\,;\overrightarrow{AD})$?
2. a. Determine the length of segment $[KI]$.
   b. Give the coordinates of the point I in the considered coordinate system.
3. Assuming the length LJ measures $\frac{\sqrt{3}}{2}$, give the coordinates of the point J.
4. Consider the line (d) of equation:
   $y=\big(\sqrt{3}-2\big)x+1$
   Show that the three points D, I, and J belong to the line (d).

21. *Deepening: decomposition of vectors in any basis*

E.8112 Consider a rectangle $ABCD$ and the three points E, F, G defined by:
$\overrightarrow{AE}=\frac{1}{2}\cdot\overrightarrow{AB}$; $\overrightarrow{DF}=\frac{2}{3}\cdot\overrightarrow{DC}$; $\overrightarrow{DG}=4\cdot\overrightarrow{DA}$



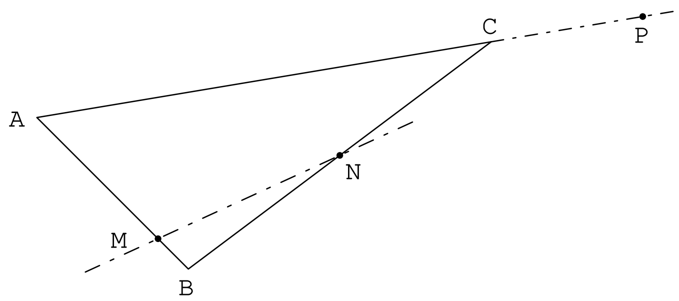
Consider the two vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$ non-colinear forming a vector basis.

(1) Determine the decomposition of the vector $\overrightarrow{EF}$ in the base $\left(\overrightarrow{AB}\,;\overrightarrow{AD}\right)$.

(2) Establish the decomposition of the vector $\overrightarrow{EG}$ in the base $\left(\overrightarrow{AB}\,;\overrightarrow{AD}\right)$:
$$\overrightarrow{EG} = -\frac{1}{2}\cdot\overrightarrow{AB} - 3\cdot\overrightarrow{AD}$$

(3) Demonstrate that the points E, F and G are aligned.

E.6664 In the plane, consider the triangle ABC shown below:



The points M, N and P are defined by the relations:
$$\overrightarrow{AM} = \frac{4}{5}\cdot\overrightarrow{AB} \quad ; \quad \overrightarrow{BN} = \frac{1}{2}\cdot\overrightarrow{BC} \quad ; \quad \overrightarrow{AP} = \frac{4}{3}\cdot\overrightarrow{AC}$$

The study will be carried out in the reference frame $\left(B\,;\overrightarrow{BA}\,;\overrightarrow{BC}\right)$.

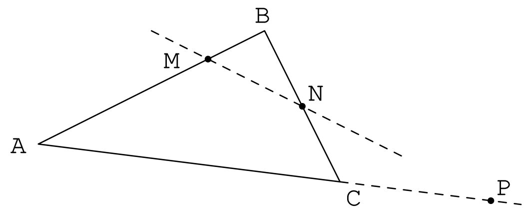
(1) Give the coordinates of points M and N.

(2) (a) Determine the coordinates of the vector $\overrightarrow{AC}$.

(b) Deduce the coordinates of the point P.

(3) Justify that the points M, N and P are aligned.

E.5342 In the plane, consider the triangle ABC:



Consider the points M and N defined by:
$$\overrightarrow{BM} = \frac{1}{4}\cdot\overrightarrow{BA} \quad ; \quad \overrightarrow{BN} = \frac{1}{2}\cdot\overrightarrow{BC}$$

We define the point P by the vector relation:
$$\overrightarrow{AP} = \alpha\cdot\overrightarrow{AC} \quad \text{où } \alpha\in\mathbb{R}$$

(1) Express $\overrightarrow{AC}$ in terms of the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

(2) The plane is given the reference frame $\left(B\,;\overrightarrow{BA}\,;\overrightarrow{BC}\right)$:

(a) Determine the coordinates of the vector $\overrightarrow{MN}$ and the vector $\overrightarrow{MP}$ as a function of the real α.

(b) Determine the value of α so that the points M, N and P are aligned.