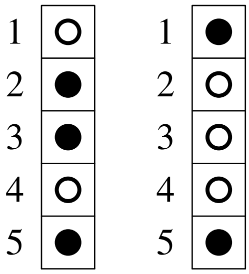
1. *Logic*

E.8342

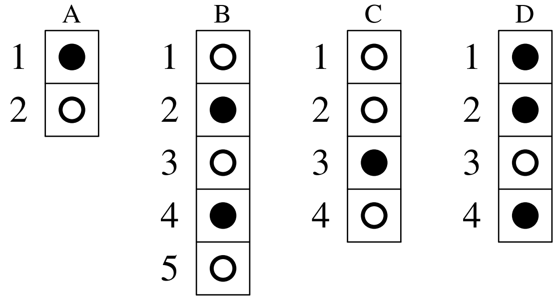
There are n counters vertically. They are black on one side, white on the other, and are numbered from 1 to n. At the start of the game, each pawn randomly presents its black or white face. **At each move - which we call a *operation* throughout the following - one of the pawns and all its neighbors on top** are turned over.



The drawing opposite shows an example of the change made to an initial configuration by an operation with the third token.

The aim of the game is to find a sequence of operations such that all pawns show their white face.

1. Does the order of two operations matter?
2. What is the combined effect of two identical operations?
3. Indicate the numbers of the pawns to be turned over to see only white faces, in the situations shown below.



E.8327

A faulty calculator only allows:

- type positive or zero numbers;
- perform the following operation: from three numbers entered successively $(x\,;y\,;z)$, it displays 0 if $x=y$ and the result of $\dfrac{z}{x-y}$ otherwise.

$$(x\,;y\,;z) \longrightarrow \begin{cases} \dfrac{z}{x-y} & \text{if } x\neq y \\ 0 & \text{si } x=y \end{cases}$$

- the use of parentheses to compose calculations.

1. By detailing the calculations, check the following results given by the calculator:
$$(0\,;1\,;2) \longmapsto -2 \quad ; \quad \Big((2\,;0\,;1)\,;1\,;1\Big) \longmapsto -2$$
2. What gives $(2\,;0\,;1)$, $(0\,;2\,;1)$ et $\big(2\,;1\,;\,(2\,;1\,;2)\big)$?
3. Give a calculation to obtain -1.
4. Verify that the calculation $(a\,;0\,;1)$ yields the inverse of a for any $a>0$.

2. *Algebra*

E.8347

For all natural numbers m and n, we call the triangle of m by n, and we note $m\Delta n$, the number defined by the following rules, which we admit are possible:

- $0\Delta n = n+1$
- $n\Delta 0 = (n-1)\Delta 1$ as soon as $n\neq 0$;
- $(n+1)\Delta(m+1) = n\Delta\big((n+1)\Delta m\big)$

Warning, $m\Delta n$ is not necessarily equal to $n\Delta m$.

Some results

1. a. Show that: $1\Delta 0=2$ et $1\Delta 1=3$
   b. Calculate $1\Delta 2$
   c. More generally, determine, for any natural number n, the value of $1\Delta n$. We can pose $u_n=1\Delta n$ and check that the sequence (u_n) is arithmetic.
2. a. Calculate $2\Delta 0$, $2\Delta 1$ and $2\Delta 2$.
   b. Justify, that for any natural number n:
   $2\Delta n=2n+3$
3. a. Calculate $3\Delta 0$, $3\Delta 1$ and $3\Delta 2$.
   b. Demonstrate that, for any natural number n, $3\Delta n$ is equal to $2^{n+3}-3$. We can pose $v_n=3\Delta n$ and show that, for any n greater than or equal to 1:
   $v_n = 2{\cdot}v_{n-1}+3$.

Illustration from $3\Delta n$

One artist illustrated the values $3\Delta 0$ and $3\Delta 1$ in this way:

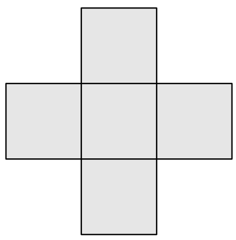


Figure 1

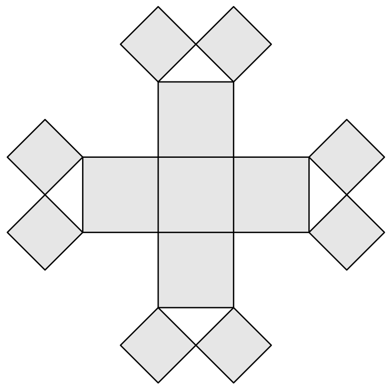


Figure 2

4. Draw on the copy a third figure that would logically complete this sequence of drawings and illustrate the value of $3\Delta 2$.
5. Suppose the side of a square in figure 1 measures 1 cm.
   a. Determine the respective areas of figures 1 and 2.
   b. What would be the area of the figure illustrating $3\Delta n$? Any overlaps will be ignored.

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E.8348 It is assumed that there exists a function f defined on the set of natural integers $\mathbb{N}$ verifying the property :

(E) : for all x and y of $\mathbb{N}$, $f(x+y)=f(x)\cdot f(y)-x\cdot y$

Preliminary

Show that $f(0)=1$. The results can be admitted in the following parts.

A. Study of a first example :

We assume here that : $f(1)=3$.

1. Calculate $f(2)$ then $f(3)$.
2. Show by two separate calculations that $f(4)=60$ and that $f(4)=63$. Conclude.

B. Study of a second example :

We assume here that : $f(1)=0$.

1. Calculate $f(2)$, $f(3)$ and $f(4)$.
2. Conjecture the expression of $f(n)$ as a function of n.
3. Demonstrate this conjecture.
4. Prove that for the function found at 2 and 3 the property (E) is indeed verified.

C. General case

First part : we note $f(1)=a$

1. Express $f(2)$ and $f(3)$ in terms of a.
2. Express $f(4)$ as a function of a in two different ways.
3. Deduce that : $a=0$ or $a=2$.

Second part : we study the second case :

It is assumed that : $f(1)=2$.

Express $f(n)$ as a function of n.

3. *Arithmetic*

E.8326 A non-zero natural number is a Harshad number if it is divisible by the sum of its digits.

For example, $n=24$ is a Harshad number because the sum of its digits is $2+4=6$, and 24 is indeed divisible by 6.

1. a. Show that 364 is a Harshad number.
   b. What is the smallest integer that is not a Harshad number?
2. a. Give a Harshad number of 4 digits.
   b. Let n be a non-zero integer. Give a Harshad number of n digits.

E.8338 Here is an algorithm applicable to three-digit integers whose hundreds digit is not equal to the units digit :

Step 1 : Inverser the order of the digits *(e.g.* 275 *becomes* 572*)*
Step 2 : Calculate the difference between the larger and smaller of these two numbers.
Step 3 : Reiterate step 1 on the number obtained.
Step 4 : Adding these last two nombres

1. Apply the algorithm to numbers 123, 448 and 946.
2. What can we conjecture?

E.8337 An integer is said to be *digisible* when the following three conditions are verified :

- none of its numbers is zero ;
- it is written with all different digits ;
- it is divisible by each of them.

For example,

- 24 is *digisible* because it is divisible by 2 and by 4.
- 324 is *digisible* because it is divisible by 3, by 2 and by 4.
- 32 is not *digisible* because it is not divisible by 3.

Recall that an integer is divisible by 3 if, and only if, the sum of its digits is divisible by 3.

1. Suggest another number *digisible* with two digits.
2. a. Give all single-digit factors of the number 1000.
   b. Deduce a four-digit *digisible* number.
3. Let n be an integer *digisible* written with a 5.
   a. Demonstrate that 5 is the digit of its units.
   b. Demonstrate that all the digits of n are odd.

E.8332 We start with a strictly positive integer n :

- If n is even, we transform it into $\dfrac{n}{2}$
- If n is odd $(n>1)$, we turn it into $3n+1$.
- If $n=1$, we stop.

Examples :

- If $n=6$, we get the sequence :
  $6\longmapsto3\longmapsto10\longmapsto5\longmapsto16\longmapsto8\longmapsto4\longmapsto2\longmapsto1$
- If $n=13$, we get the sequence :
  $13\mapsto40\mapsto20\mapsto10\mapsto5\mapsto16\mapsto8\mapsto4\mapsto2\mapsto1$

It has been observed using a computer program, that for every integer tested, the sequence always results in 1. But this result has not yet been demonstrated.

We can also be interested in the length of this sequence, which we'll denote $L(n)$.

For example : $L(6)=9$ and $L(13)=10$.

1. Determine $L(n)$ for integers from 1 to 12.
2. Let p be an integer, consider the integer $n=2^p$. Express $L(n)$ as a function of p.
3. Find an integer n between 2^{2008} and 2^{2009} such that : $L(n)=2012$.
   Hint : We could look for a number of the form $2^p\times q$.

4. *Arithmetic and congruence*

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E.8329

1 a Starting from 12 589 and counting from 29 to 29, can we reach the number 12 705?

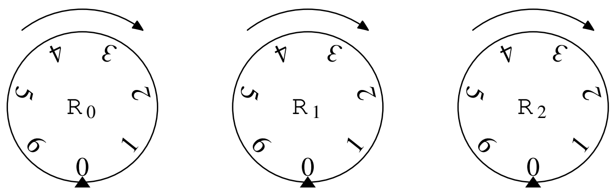
b Starting from 1 485 and counting from 29 in 29, can we reach the number 310 190?
Explain your approach.

2 What is the smallest positive integer from which, counting from 29 to 29, we can reach 2013?

3 Are there positive integers less than 2 013 from which it is possible to reach this number both by counting from 29 in 29 and by counting from 31 in 31?
If so, find them all.

E.8330 A counter consists of three toothed wheels, named R_0, R_1 and R_2 each comprising 7 notches, numbered from 0 to 6. This counter is designed so that:

- The wheels always turn from one notch to the next, in this order:
  $0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 5 \longrightarrow 6 \longrightarrow 0$
- When the wheel R_0 makes a full turn, i.e. when it turns 7 notches then the wheel R_1 turns one notch.
- When the wheel R_1 makes one complete turn, i.e. when it turns 7 notches, then the wheel R_2 turns one notch.
- Initially, the wheels R_0, R_1 and R_2 all display 0.



Between each question, the counter is reset, i.e. each wheel displays 0 *again*

1 We turn the wheel R_0 by 15 notches. What are the numbers displayed by the wheels?

2 We turn the wheel R_0 by 100 notches. What are the numbers displayed by the wheels?

3 We turn the wheel R_0 until the wheel R_2 displays 5 for the first time. How many notches has R_0 been turned?

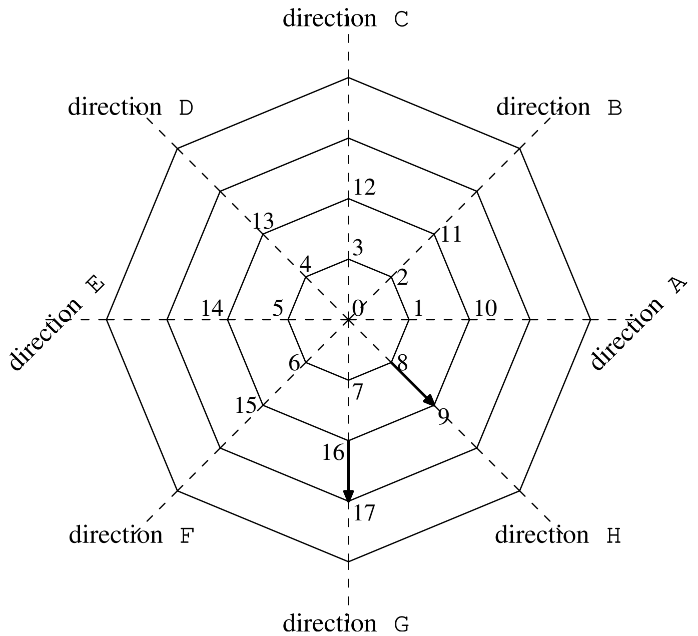
4 How many notches must R_0 be turned for the wheels to return to 0 at the same time for the first time?

5 We turn the wheel R_0 by 3580 notches. What are the numbers displayed by the wheels then?

E.8331 Consider regular octagons, of the same center O.
On the vertices of the central octagon, note the first eight non-zero integers.
On the vertices of the second octagon, we inscribe the next 8 first integers, with a rotation of 45 degrees around the point O.
And so on...

Each integer is said to have a direction *(A, B, C, D, E, F, G or H relative to the origin O).*
For example, 1 has direction A, 2 has direction B...

Here's a figure representing the first four octagons:



1 What will be the first integer inscribed on the fourth octagon? Specify its direction.

2 Determine the first integer inscribed on the eighth octagon. Specify its direction.

5. *Arithmetic and function*

E.8340 For all natural numbers m and n, we call the triangle of m by n, and we note $m\Delta n$, the number defined by the following rules, which we admit are possible:

- $0\Delta n = n+1$
- $n\Delta 0 = (n-1)\Delta 1$ as soon as $n \neq 0$;
- $(n+1)\Delta(m+1) = n\Delta\big((n+1)\Delta m\big)$

Warning, $m\Delta n$ is not necessarily equal to $n\Delta m$.

Some results

1 a Show that: $1\Delta 0 = 2$ et $1\Delta 1 = 3$

b Calculate $1\Delta 2$

c Assume that $1\Delta n = n+2$. Show that:
$1\Delta(n+1) = n+3$

2 a Calculate $2\Delta 0$, $2\Delta 1$ and $2\Delta 2$.

b Assume that $2\Delta n = 2n+3$. Show that:
$2\Delta(n+1) = 2n+5$

3 a Calculate $3\Delta 0$, $3\Delta 1$ and $3\Delta 2$.

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E.8339 It is assumed that there exists a function f defined on the set of natural integers $\mathbb{N}$ verifying the property:

(E): for all x and y of $\mathbb{N}$, $f(x+y)=f(x)\cdot f(y)-x\cdot y$

Preliminary

Show that $f(0)=1$. The results can be admitted in the following parts.

A. Study of a first example:
We assume here that: $f(1)=3$.

1. Calculate $f(2)$ then $f(3)$.
2. Show by two separate calculations that $f(4)=60$ and that $f(4)=63$. Conclude.

B. Study of a second example:
We assume here that: $f(1)=0$.

1. Calculate $f(2)$, $f(3)$ and $f(4)$.
2. Conjecture the expression of $f(n)$ as a function of n.
3. Demonstrate this conjecture.
4. Prove that for the function found at (2) and (3) the property (E) holds.

6. Geometry

E.8341 An artist has created the two figures below:

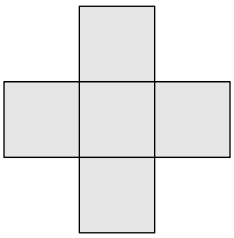


Figure 1

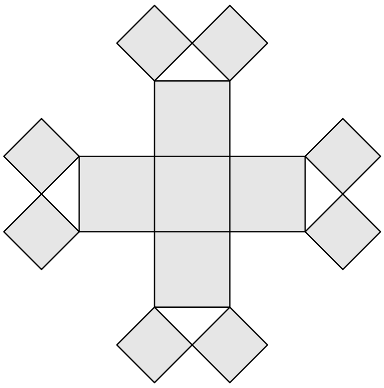


Figure 2

1. The side of a square in figure 1 measures $1\,cm$. Determine the area of the figure 1.
2. Figure 2 was constructed from figure 1 to which eight identical squares were replayed. Determine the area of figure 2.

E.8328 **Angle measurements approximately**

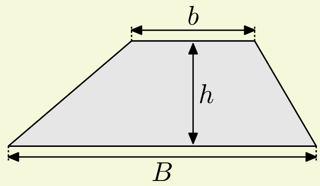
A triangle ABC is said to be approximately right-angled at a vertex A if the measure of the angle at A is in the interval $[75^o\,;105^o]$. A triangle ABC is said to be approximately isosceles at a vertex A if the measures of the angles at B and C differ by 15^o at most.

1. (a) Is a right-angled triangle approximately right-angled? Is an isosceles triangle approximately isosceles.
   (b) Can a triangle be right-angled in two vertices? Approximately right-angled in two vertices? If so, when it is additionally acutangle *(i.e. all its angles are acute)*, is it approximately isosceles?
2. Is there an acutangular triangle that is neither approximately right-angled nor approximately isosceles?
3. Write a program *(in natural language or calculator)*, to be copied onto your copy, testing whether a triangle ABC whose three angles A, B and C are known is approximately isosceles.

E.8335

Reminder: Area of a trapezoid

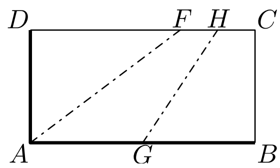
$$A=\frac{(B+b)\times h}{2}$$



A rectangular pizza $ABCD$ has crust on two consecutive sides, $[DA]$ and $[AB]$. **We want to divide the pizza into three equal pieces**: each piece must have the same length of crust and the same area.

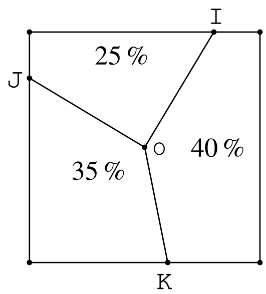
In each situation, we fix the length of the short side $AD=1$.

In the specific case shown opposite, we assume that the division is equal.
What is the length AB?
Determine the lengths:
DF, FH, and HC.



E.8336

In a square of side $10\,cm$, we want to make a statistical graph in which the areas of the 3 parts must be proportional to the frequencies they represent *(O is the center of the square)*.
The point I is $2\,cm$ from the nearest vertex.



Calculate the distances of J and K to the nearest vertices of the square.

7. Geometry and Pythagorean theorem

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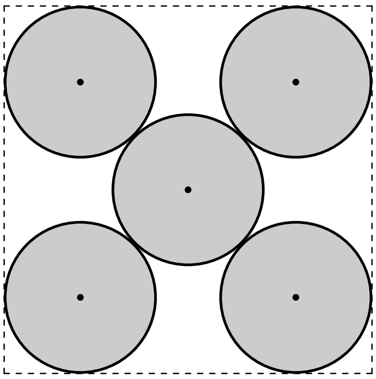
E.8344

Five circles of radius $1\,cm$ have been placed in a square as shown in the drawing.

The circles are tangent to each other and tangent to the sides of the square.

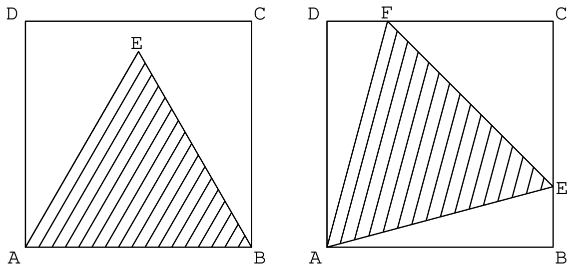
Determine the length of one side of the square.



E.8334 The metal workshop of a shipyard cuts parts of various shapes from square steel plates that it orders from the rolling mill.
To limit material losses and therefore production costs, the shop foreman must determine in advance the size of the square plates he needs to order according to the parts to be cut.
For some orders, only the shape and surface of the parts to be cut are transmitted to them.
In each of the following three parts, the cutting of certain types of parts is studied.
These parts can be treated independently of each other.
Where necessary, lengths should be rounded to the nearest mm, and areas to the nearest cm^2.

Part 1 : Cutting triangular parts

The workshop needs to produce a part that has the shape of an equilateral triangle with a surface area of $20\,m^2$.
The workshop manager is considering two cutting solutions as illustrated in the following diagrams :



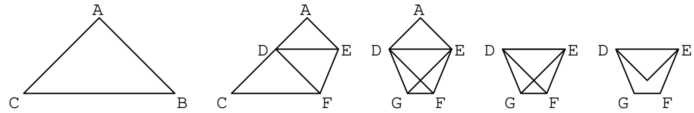
Note a the side of the triangle and c the side of the square.

1. **Schematic n^{o}1 :**
   a. Express the height h as a function of a.
   b. Deduce the side a of the square to be constructed to meet the constraints.
   c. Calculate the area of steel lost with this method.
2. **Diagram n^{o}2 :**
   a. Justify that the angle $\angle BAE$ measures 15^o.

E.8353 In this exercise, we consider only A4-sized sheets of paper, i.e. with dimensions $21\,cm$ and $29.7\,cm$.

Part A - Creating a cornet by folding

Using an A4 sheet of paper, create a cornet by successive cutting and folding.
Here's the protocol for constructing this cornet, illustrated in the figure below :



Step 1 : cut out a square $ABA'C$ side $21\,cm$ from the A4 sheet of paper.

Step 2 : fold this square according to (BC) to obtain an isosceles right-angled triangle verifying : $AB = AC = 21\,cm$

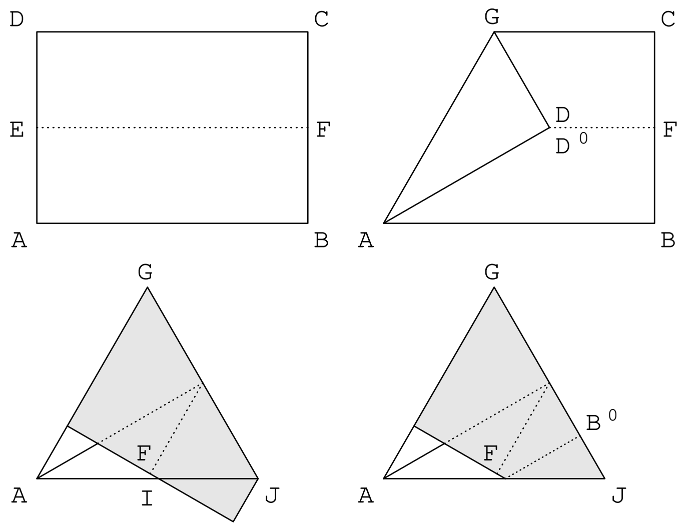
Step 3 : place the vertex B on a point D of the segment $[AB]$ such that, after folding, the line (DE) is parallel to the line (BC).

Step 4 : perform a similar procedure starting from vertex B.

Step 5 : fold the two top triangles, one in front, one behind.

When we measure AD, we get $8.7\,cm$ *(rounded to the nearest mm)*.
What is the exact value of the distance AD?

Part B - Making a particular triangle y Using a sheet of A4 paper, we now need to make a particular triangle by successive folds. This sheet is modeled by a rectangle $ABCD$. E and F denote the middles of segments $[AD]$ and $[BC]$. respectively Here's the protocol for constructing this triangle illustrated in the figure below.



The rectangle in the following figure corresponds to the unfolded A4 sheet after the triangle has been constructed. We wish to determine the nature of the triangle thus constructed.

1. Show that the measure of the angle $\angle GAJ$ is equal to 60^o.
2. Show that we fold the point C onto the segment $[AG]$.
3. Show that the point B is folded over the segment $[JG]$. Conclude.

8. *Geometry, Pythagoras and Thales theorem*

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E.8333 A and B are two points on a circle with center O and radius 5 such that $AB=6$.

The square $PQRS$ is inscribed in the angular sector OAB so that :

- P is on radius $[OA]$;
- S is on radius $[OB]$;
- Q and R are two points on the arc of a circle connecting A and B.

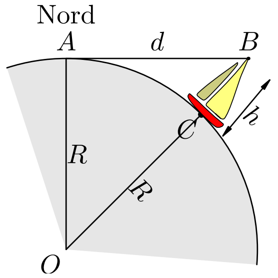
(1) Make a figure corresponding to the proposed situation.

(2) Calculate the area of the square $PQRS$.

9. *Geometry and trigonometry*

E.8352 **Calculating the radius R of the Earth**

(1) An observer is standing on the shore at A and sees a boat moving away to the south, the height of which is known h above the waterline *(as shown in the figure below, which is not to scale)*. Due to the roundness of the Earth, the ship disappears completely from the horizon once it is $d=AB$ away from the coast as the crow flies.



We ignore the size of the observer so that $OA=R$.

We assume that the distance d is known. We recall that the horizon line (AB) is tangent to the Earth at A. Find a relationship between R, d, and h.

(2) Justify that $\dfrac{h}{R}$ is very small.

(3) Deduce that : $R \approx \dfrac{d^2}{2{\cdot}h}$

(4) Numerical application : for $h=20\,m$ and $d=16\,km$, give a value for the radius R of the Earth, rounded to the nearest kilometer.

Horizon line

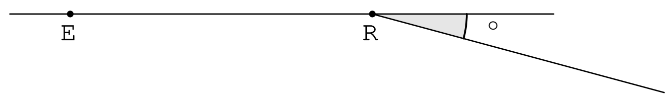
Throughout this section, we agree that $R=6400\,km$.

(5) We are looking for the height h of the boat in the figure above so that the lookout, located at the top of the mast, can see the shore of an island located at a given distance *(in a straight line)* d.

Show that this amounts to solving, for a given d, the equation with unknown h : $\quad (h+R)^2 = d^2 + R^2$

(6) In the case where $d=50\,km$, calculate the height h and the length of the arc $\overset{\frown}{AH}$.

E.9549 Pierre and his daughter Eloise are walking along a horizontal road. At a point R, this road descends making an angle θ of 5^o with the horizontal *(see figure)*

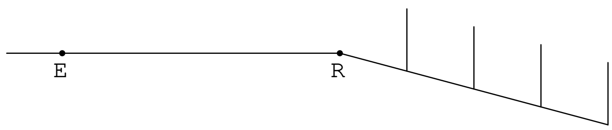


Eloise, whose eyes are 1.6 meter from the ground, stops at a point E, 24 meters from the R point.

His father continues walking, passes the point R then enters the sloping part of the road.

(1) When he is 86 meters from R, he disappears from his daughter's view.
Determine Pierre's height.

(2) On the sloping part of the road, posts 6.5 meters high are planted vertically every 28 meters, as in the diagram below



The foot of the first post is 28 meters from the point R. It is assumed that the posts cannot hide from each other.

How many poles can Eloise see from where she is?

(3) What is, in reality, the measure of the angle θ, given that Eloise can only see 5 poles?

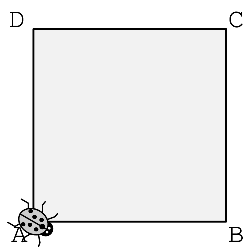
We can use the formula : $\quad 1+(\tan\theta)^2 = \dfrac{1}{(\cos\theta)^2}$

We'll give an approximate value of θ to the nearest 10^{-3}.

10. *Probability*

E.8349

A ladybug moves along the sides of a square $ABCD$ starting from point A. She can walk backwards if she wishes. Any path taken by the ladybug along a side of the square is called a displacement. A walk is made up of displacements, thus :



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$A \longrightarrow B \longrightarrow A \longrightarrow D \longrightarrow C$
is a walk of four moves whose arrival is point C.

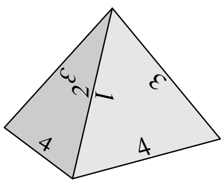
In this part, the ladybug moves randomly along the sides of a square $ABCD$ and all its moves are considered equiprobable.

1. a. Can the ladybug reach the point B in three moves?
   b. What are the possible arrivals for a three-move walk?
   c. What are the possible arrivals if the walk has an even number of moves?
   d. What are the possible arrivals if the walk has an odd number of moves?
2. In this question, the ladybird makes two moves.
   Possibly using a tree, calculate the probability of the event A_2 : "*the ladybug arrives in A by making two déplacements*".
3. Reproduce and complete the table below :

Nombre de déplacements de la marche	1	2	3	4	5
Probabilité que la coccinelle arrive en A					

E.8350

A tetrahedral die has four faces like the one shown opposite. When such a die is rolled, the result is the number inscribed closest to the tetrahedral base. In our example, the tetrahedral die fell on face 4.



Antoine, Baptiste, Cyril and Diane play with four regular, balanced tetrahedral dice, but which are not numbered in the usual way. Thus, Antoine's die has four sides numbered 1, 6, 6 and 6. With this die, the number 1 is obtained with probability $\frac{1}{4}$ and the number 6 with probability $\frac{3}{4}$.

- Baptiste's die is numbered 4, 4, 5 and 5 ;
- Cyril's 3, 3, 3 and 8 ;
- and finally, dianne's 2, 2, 7 and 7.

1. Each player rolls this tetrahedral die once. Who has the best chance of getting a number greater than or equal to 6?
2. Players begin a series of duels : Antoine plays Baptiste, Baptiste plays Cyril, Cyril plays Diane, Diane plays Antoine. The winner of each duel is the player with the highest score.
   a. Show that in the first duel Antoine wins against Baptiste with probability $\frac{3}{4}$.
   b. Give the players' winning probabilities in the other three duels.
3. Antoine, Baptiste, Cyril and Diane simultaneously throw their dice. The player with the highest number wins.
   a. Show that the probability of Baptiste winning is equal to $\frac{3}{32}$.
   b. Who has the best chance of winning this game?

11. *Out of program*

E.8343 Let n be a natural number greater than or equal to 2. There is an urn containing n balls that can be of different colors.
The game consists of randomly extracting a ball from the urn, then without returning it to the urn extracting a second ball from the urn. The player has won when the two balls drawn are the same color.
It is assumed that on each draw, all the balls in the urn have the same probability of being drawn.
The game is said to be fair when the probability $P(G)$ that the player wins is equal to $\frac{1}{2}$.

1. a. Demonstrate that if the urn contains 10 balls of which 4 are white and 6 are red then $\mathcal{P}(G) = \frac{7}{15}$.
   b. Calculate $\mathcal{P}(G)$ when the urn contains 12 balls including 4 white, 6 red and 2 black.
2. In this question, the urn contains 6 red balls and other balls that are all white.
   a. Let x be the number of white balls contained in the urn.
   Show that : $\mathcal{P}(G) = \dfrac{x(x-1)+30}{(x+6)(x+5)}$

12. *Unclassified exercises*

E.8354 From two positive integers, we construct a list of numbers où each number is the sum of the previous two.

1. Choose two positive integers less than 10 and determine the first ten numbers from the list defined above.
2. A mathemagician claims to be able to quickly and exactly determine the sum of the first ten numbers of any list constructed in this way. Show that, whatever the starting numbers, this sum is a multiple of one of the numbers in the list whose position will be determined.

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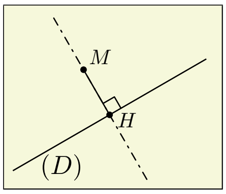
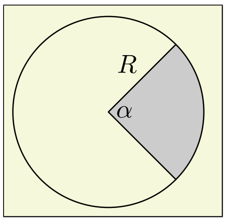
E.8355

Definition : The ***distance between a point*** M ***and a line*** (D) is called the distance MH, where H is the point of intersection of (D) with the line perpendicular to (D) passing through M.



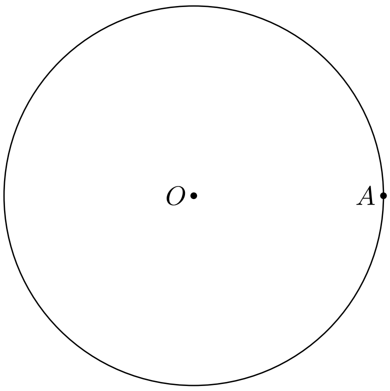
In the figure opposite, if the radius of the disk is R, and if the angle of the shaded sector measures α *(in degrees)*, then the area of the shaded portion of the disk is : $\dfrac{\pi\cdot\alpha\cdot R^2}{360}$.

In part **2** of the exercise, we will consider the distance from point M to segment $[BC]$ to be the distance from point M to line (BC).



Part 1

Let $\mathcal{C}$ be a circle with center O, A a point on this circle, and $\mathcal{D}$ the disk bounded by this circle.



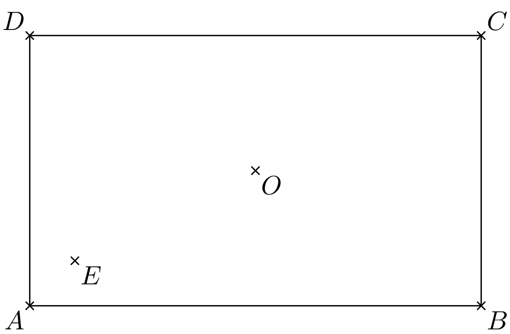
1. Reproduce the figure and represent the set of points on the disk that are equidistant from O and A.

2. Shade the set of points on the disk that are closer to O than to A.

3. Let M be a point chosen at random with equal probability on the surface of the disk $\mathcal{D}$.
What is the probability that M is closer to O than to A?

Part 2

Let $ABCD$ be a rectangle with length $AB=20\,cm$ and width $BC=12\,cm$, centered at O
Let E be a point located inside the rectangle, close to A, at $2\,cm$ from each edge *(as shown in the figure below, which is not to scale)*.
Let M be a point determined randomly and equally likely within the rectangle $ABCD$.



1. What is the probability that M is closer to side $[BC]$ than to side $[AD]$?

2. a. Reproduce the rectangle and represent all the points inside the rectangle that are equidistant from sides $[AB]$ and $[BC]$.

   b. Shade the set of points inside the rectangle that are closer to side $[BC]$ than to side $[AB]$.

   c. What is the probability that M is closer to side $[BC]$ than to side $[AB]$?

3. What is the probability that M is closer to side $[AB]$ than to sides $[BC]$, $[CD]$, and $[DA]$?

4. What is the probability that M is closer to O than to E?

5. What is the probability that M is closer to O than to the four vertices A, B, C, and D?

E.8345

Three distinct natural numbers a, b, c **arranged in strictly croissant** order, $a<b<c$, are in arithmetic progression if : $c-b=b-a$

We then say that $(a\,;b\,;c)$ is an arithmetic triplet.

1. Complete the following arithmetic triplets :

   a. $(57\,;101\,;\ldots)$ b. $(57\,;\ldots\,;101)$

   c. $(\ldots\,;57\,;101)$

2. a. Can we find an arithmetic triplet $(a\,;b\,;c)$ whose sum is 2012?

   b. How many arithmetic triplets are there $(a\,;b\,;c)$ with sum 2013?

3. We randomly take three integers a, b, c in 1, 2, 3, ..., 10 with $a<b<c$. What is the probability that $(a\,;b\,;c)$ is an arithmetic triplet?

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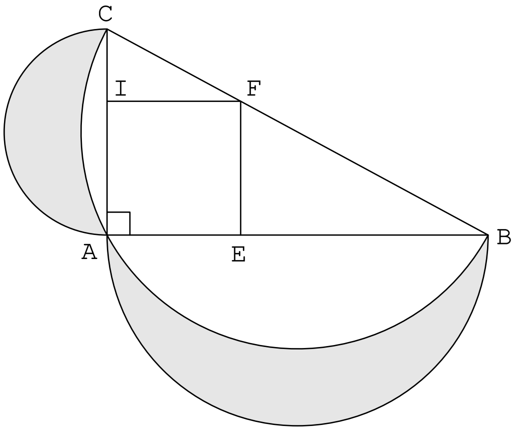
E.8346 An association wants to create a logo.

This logo has been designed from the following construction :

- ABC is a right-angled triangle in A,
- we pose : $AC=x$; $AB=y$; $BC=z$,
- we drew the semicircles of diameters $[AB]$, $[AC]$, $[BC]$ and the square $AEFI$ such that $E\in[AB)$, $F\in[BC)$ and $I\in[AC)$.

1. For this question we consider the following figure :

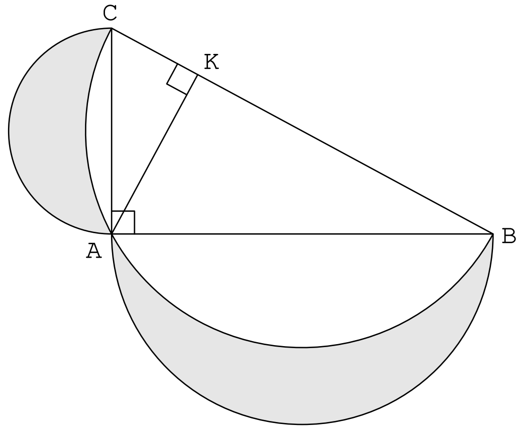


a. Calculate the length of the side of the square $AEFI$ as a function of x and y.

b. What can be said about the point I if the triangle ABC is isosceles? *(justify)*

c. We assume $y=4$. Can the area of the square $AEFI$ be equal to 9? *(justify)*

2. Let K be the foot of the height from A of the triangle ABC. We pose $AK=h$. We therefore have the following figure :



a. Justify that : $(x+y)^2=z^2+2\cdot z\cdot h$.

b. Similarly express $(x-y)^2$ as a function of z and h. Show that h is less than half of z.

c. Is it possible that : $z=10$ and $h=4.8$? If yes, determine the values of x and y.

3. Compare the area of the triangle ABC to the area of the shaded surface.

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