1. *Solving systems*

E.6348 Solve the following system:

$$\begin{cases} x + 2y - z = -2 \\ 3x + y + 2z = -1 \\ x - y + 3z = 3 \end{cases}$$

(It will be shown that this system admits a single triplet solution).

E.8653 Solve the following system:

$$\begin{cases} 3x + y - z = 1 \\ x - y + 2z = 2 \\ -x + 5y - 9z = -5 \end{cases}$$

(It will be shown that this system admits no solution)

E.8654 Solve the following system:

$$\begin{cases} x + y - z = 5 \\ 2x - y + 4z = -2 \\ 5x - y + 7z = 1 \end{cases}$$

(We will show that this system admits an infinite number of solutions which we will write in the form $(\dots;\dots;z)$ où $z\in\mathbb{R}$)

2. *Parametric representations of a straight line*

E.5400 Give a parametric representation of the line (d) passing through the point A and admitting the vector $\vec{u}$ as directing vector in each case below:

ⓐ $A(3;0;-2) \quad ; \quad \vec{u}(-1;-2;1)$

ⓑ $A(2;-1;1) \quad ; \quad \vec{u}(2;0;-4)$

E.4038 ⚠ Space is provided with an orthonormal reference frame $(O;\vec{i};\vec{j};\vec{k})$.

Consider the point A with coordinates $(-2;8;4)$ and the vector $\vec{u}$ with coordinates $(1;5;-1)$.

Determine a parametric representation of the line (d) passing through A and with direction vector $\vec{u}$.

E.4035 ⚠ *Indicate whether the following statement is true or false, giving reasons for your answer.*

The space is related to the orthonormal reference frame $(O;\vec{i};\vec{j};\vec{k})$.

The line in space passing through the point B with coordinates $(2;3;4)$ and with vector $\vec{u}(1;2;3)$ as its direction vector has the parametric representation:

$$\begin{cases} x = t + 1 \\ y = 2t + 1 \\ z = 3t + 1 \end{cases} \quad \text{where } t\in\mathbb{R}$$

3. *Parametric representations of lines and relative positions*

E.6347 Consider the space provided with a $(O;\vec{i};\vec{j};\vec{k})$ and the straight line (d) admitting the parametric representation:

$$\begin{cases} x = -1 + t \\ y = 1 + 2t \\ z = 2 - 3t \end{cases} \quad , t\in\mathbb{R}$$

① ⓐ Show that the point $A\left(\frac{1}{2};4;-\frac{5}{2}\right)$ belongs to the straight line (d).

ⓑ Show that the point $B\left(-\frac{9}{4};\frac{3}{2};-\frac{23}{4}\right)$ does not belong to the line (d).

② Consider the straight line (d') admitting the parametric representation:

$$\begin{cases} x = 1 + \frac{3}{2}\cdot t \\ y = 3 - 3t \\ z = -4 - \frac{9}{2}\cdot t \end{cases} \quad , \text{où } t\in\mathbb{R}$$

ⓐ Show that the point A belongs to the line (d').

ⓑ What is the relative position of the straight lines (d) and (d')?

E.5401 Consider the space provided with a $(O;\vec{i};\vec{j};\vec{k})$ and the two straight lines (d) and (d') admitting as parametric representations:

$$(d): \begin{cases} x = 3 + 2\cdot t \\ y = -1 - 2\cdot t \\ z = 2 + 6\cdot t \end{cases} \quad (d'): \begin{cases} x = -1 - t \\ y = -1 + t \\ z = -3\cdot t \end{cases} \quad \text{où } t\in\mathbb{R}$$

① Show that the straight lines (d) and (d') are parallel.

② Show that the straight lines (d) and (d') are parallel-strict?
(it will be shown that a point of (d) does not belong to (d'))

E.5413 In space provided with a $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$ orthonormal, consider the two points M and N of coordinates:

$M(-1\,;2\,;3)$; $N(1\,;-2\,;9)$

(d): $\begin{cases} x=-1+\ t \\ y=\ \ 2-2t \\ z=\ \ 3+9t \end{cases}$ (d'): $\begin{cases} x=\ \ 3+2t \\ y=\text{-}6-\ t \\ z=12-2t \end{cases}$ (d''): $\begin{cases} x=-3+\ t \\ y=\ \ 6-2t \\ z=\text{-}3+3t \end{cases}$

où t is a real traversing the set of reals.

1. a. To which straight lines does the point M belong?
   b. Which of the straight lines (d), (d') and (d'') represents the straight line (MN)?
2. Determine, by another method, another representation of the line (MN).

E.4057 ⚠ Space is referred to an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$. Let A be the coordinate point $(3\,;1\,;3)$.

Consider the line $\mathcal{D}$ in space passing through A and with direction vector $\vec{u}\,(1\,;2\,;-1)$ and the line $\mathcal{D}'$ of parametric equations:

$$\begin{cases} x = 3 + 2t \\ y = 3 + \ t \\ z = \ \ \ \ \ \ \ t \end{cases} \text{ où } t\in\mathbb{R}$$

Say which of the following three statements is correct. No justification is required:

1. The straight lines $\mathcal{D}$ and $\mathcal{D}'$ are coplanar and parallel;
2. The straight lines $\mathcal{D}$ and $\mathcal{D}'$ are coplanar and secant;
3. The straight lines $\mathcal{D}$ and $\mathcal{D}'$ are non-coplanar.

E.4067 ⚠ Consider the two straight lines (d) and (d') whose parametric representations are respectively:

$$\begin{cases} x=\ \ 2+4{\cdot}t \\ y=\ \ 1-2{\cdot}t \\ z=-3+\ \ t \end{cases} \text{ où } t\in\mathbb{R} \qquad \begin{cases} x=\ \ 5-2{\cdot}k \\ y=-1+\ \ k \\ z=\ \ 2-\dfrac{1}{2}{\cdot}k \end{cases} \text{ où } k\in\mathbb{R}.$$

Show that the straight lines (d) and (d') are coplanar.

4. *Parametric representations of a line and solving systems*

E.5402 Consider the space provided with a $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$ and the two straight lines (d) and (d') admitting as parametric representation:

$$(d):\begin{cases} x=\ \ \ \ \ -t \\ y=-1-\ \ t\ ;\, t\in\mathbb{R} \\ z=\ \ 5+3t \end{cases} \qquad (d'):\begin{cases} x=\ \ 4+t \\ y=\ \ 3+t\ ;\, t\in\mathbb{R} \\ z=-1 \end{cases}$$

Show that the straight lines (d) and (d') are coplanar and secant. Specify the coordinates of the point of intersection.

E.5409 In space provided with a $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$, consider the two straight lines (d) and (d') admitting as parametric representations:

$$(d):\begin{cases} x = 1 + t \\ y = 1 + 3t \ ,t\in\mathbb{R} \\ z = \ \ \ \ -t \end{cases} \qquad (d'):\begin{cases} x = \ \ 2\ \ + t \\ y = -6 - 2t \ ,t\in\mathbb{R} \\ z = \ \ 3\ \ + t \end{cases}$$

Show that the straight lines (d) and (d') are secant and determine the coordinates of their point of intersection.

E.5403 Consider the space provided with a $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$ and the two straight lines (d) and (d') admitting as parametric representation:

$$(d):\begin{cases} x = 1-t \\ y = 1+t \text{ où } t\in\mathbb{R} \\ z = 2+t \end{cases} \qquad (d'):\begin{cases} x = -2-t \\ y = -1-t \text{ où } t\in\mathbb{R} \\ z = -1+t \end{cases}$$

Justify that these two straight lines are non-coplanar.

E.2679 ⚠ Consider the two straight lines with respective parametric representations:

$$\begin{cases} x = \ \ 2 - 3t \\ y = \ \ 1 + \ t \\ z = -3 + 2t \end{cases} \text{ where } t\in\mathbb{R}\ ;\ \begin{cases} x = \ \ 7 + 2u \\ y = \ \ 2 + 2u \\ z = -6 - \ u \end{cases} \text{ where } u\in\mathbb{R}$$

Show that these two straight lines intersect and determine the coordinates of the point of intersection.

E.4066 ⚠ Consider the two parametric lines:

$$\begin{cases} x=\ \ 2-3t \\ y=\ \ 1+\ t \\ z=-3+2t \end{cases} \text{ où } t\in\mathbb{R} \qquad \begin{cases} x=\ \ 7+2u \\ y=\ \ 2+2u \\ z=-6-\ u \end{cases} \text{ où } u\in\mathbb{R}$$

Show that these two straight lines are secant. Give the coordinates of the point of intersection.

E.4065 ⚠ Space is provided with an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$.

Consider the straight lines (d) and (d') admitting respectively the following parametric representations:

$$\begin{cases} x=\ \ 1+2{\cdot}t \\ y=-2-3{\cdot}t \\ z=-1-\ \ t \end{cases} \text{ où } t\in\mathbb{R} \qquad \begin{cases} x=2-\ \ k \\ y=1+2{\cdot}k \\ z=\ \ \ \ \ \ k \end{cases} \text{ où } k\in\mathbb{R}.$$

Show that the straight lines (d) and (d') are not coplanar.

E.8655 Consider the straight lines (Δ) and (Δ') whose parametric representations are respectively:

$$\begin{cases} x=\ \ 3+2{\cdot}t \\ y=-3-\ \ t \\ z=\ \ 6+3{\cdot}t \end{cases} \text{ où } t\in\mathbb{R} \qquad \begin{cases} x=-1-\ \ k \\ y=\ \ 2+2{\cdot}k \\ z=\ \ 1-\ \ k \end{cases} \text{ où } k\in\mathbb{R}.$$

Show that the straight lines (Δ) and (Δ') are coplanar.

5. *Parametric representations of lines and orthogonalities*

E.5414 In space provided with a $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k}\right)$ orthonormal, consider the two straight lines (d) and (d') defined by their parametric representation:

$$(d)\begin{cases} x=-3+2t\\ y=-4+3t\\ z=3-t\end{cases}\text{ où } t\in\mathbb{R}\qquad (d')\begin{cases} x=1+2t\\ y=-3-2t\\ z=-2t\end{cases}\text{ où } t\in\mathbb{R}$$

1. Show that the straight lines (d) and (d') are orthogonal to each other.
2. Are the straight lines (d) and (d') secant? If so, specify the point of intersection.

E.6968 In space provided with a $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k}\right)$ orthonormal, consider two straight lines (d) and (d') admitting as parametric representation:

$$(d)\begin{cases} x=-3+2\cdot t\\ y=\quad -\ t\\ z=-1+\ t\end{cases}\ t\in\mathbb{R}\quad ;\quad (d')\begin{cases} x=-1-2\cdot t\\ y=\ 1+3\cdot t\\ z=\quad -\ t\end{cases}\ t\in\mathbb{R}$$

1. Show that the straight lines (d) and (d') are secant. We'll determine the coordinates of their M point of intersection.
2. a. Consider the two vectors $\overrightarrow{u}\,(2\,;-1\,;1)$ et $\overrightarrow{v}\,(-2\,;3\,;-1)$. Determine the coordinates of a vector $\overrightarrow{n}$ that is orthogonal to the two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$.
   b. Deduce a parametric representation of the line (Δ) passing through the point M and orthogonal to the two lines (d) and (d').

E.4096 *We admit that if (d) and (d') are two non-coplanar straight lines, there exists a single straight line (Δ) perpendicular to (d) and (d).*

Space is referenced to the orthonormal frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k}\right)$.

We note (d) the abscissa line and (d') the line admitting the parametric representation:

$$\begin{cases} x=\quad -\ t\\ y=3+3\cdot t\\ z=1-\ t\end{cases}\quad\text{où } t\in\mathbb{R}$$

Consider the line (Δ) common perpendicular to (d) and (d'). Prove that there exist two real b and c such that the vector: $\overrightarrow{w}=b\cdot\overrightarrow{j}+c\cdot\overrightarrow{k}$ is a director vector of (Δ).

E.6405 Consider the space provided with a reference frame $(O\,;I\,;J\,;K)$ and the two straight lines (d) and (d') admitting as parametric equation:

$$(d)\begin{cases} x=\ 3+t\\ y=-5+t\\ z=\ 4\end{cases}\text{ où } t\in\mathbb{R}\quad ;\quad (d')\begin{cases} x=\ 2+2\cdot t\\ y=\ 3+\ t\\ z=-2-\ t\end{cases}\text{ où } t\in\mathbb{R}$$

1. Show that the straight lines (d) and (d') are non-coplanar.
2. We assume the existence of a line (Δ) perpendicular to the line (d) and perpendicular to the line (d')
   a. Justify the existence of a real t such that the line (Δ) admits as parametric representation:
   $$\begin{cases} x=\ 3+t+t'\\ y=-5+t-t'\\ z=\ 4+t'\end{cases}\quad\text{où } t'\in\mathbb{R}$$
   b. Deduce a parametric equation of the line (Δ).

E.4036 Space is referred to the orthonormal reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k}\right)$.

Note $\mathcal{D}$ the abscissa line and $\mathcal{D}'$, the parametric representation line:

$$\begin{cases} x=\quad -\ t\\ y=3+3t\\ z=1-\ t\end{cases}$$

1. Justify that the straight lines $\mathcal{D}$ and $\mathcal{D}'$ are not coplanar.
2. Consider the line Δ perpendicular common to $\mathcal{D}$ and $\mathcal{D}'$. Prove that there are two real b and c such that the vector $\overrightarrow{w}=b\cdot\overrightarrow{j}+c\cdot\overrightarrow{k}$ is a director vector of Δ.

6. *Parametric representations of a plane*

E.5424 In space with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k}\right)$, consider the three points:
$A\,(2\,;0\,;-1)\quad;\quad B\,(1\,;0\,;3)\quad;\quad C\,(2\,;1\,;1)$

1. Justify that the points A, B and C define a plane.
2. By choosing $\left(\overrightarrow{AB}\,;\overrightarrow{AC}\right)$ as a pair of directing vectors, show that the plane (ABC) admits as parametric representation the following system:
   $$\begin{cases} x=-t\quad\quad +2\\ y=\quad\quad t'\\ z=4t+2t'-1\end{cases}\quad\text{où } t\in\mathbb{R},\ t'\in\mathbb{R}$$
3. Justify that the point $D\,(1\,;-2\,;-1)$ belongs to the plane (ABC).

E.5425 In space with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k}\right)$, consider the three points:
$A\,(3\,;-1\,;2)\quad;\quad B\,(3\,;1\,;1)\quad;\quad C\,(2\,;-1\,;1)$

1. Justify that the points A, B and C define a plane.
2. Determine a parametric representation of the plane (ABC).

E.6350 In space provided with a reference frame $\left(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k}\right)$, consider the three points:
$A(2\,;0\,;-1)\quad;\quad B(1\,;2\,;2)\quad;\quad C(-1\,;1\,;-2)$

1. a. Do the points A, B, C determine a plane? Justify your answer.
   b. Determine a parametric representation of the plane (ABC)
2. a. Consider the point $D\,(0\,;3\,;1)$. Does the point D belong to the plane (ABC)? Justify your answer.
   b. Consider the point $E\,(7\,;0\,;4)$. Does the point E belong to the plane (ABC)? Justify your answer.

7. *Coplanarity*

E.6945

(1) Consider the three vectors:
$\vec{u}\,(1\,;-1\,;2)$; $\vec{v}\,(1\,;1\,;3)$; $\vec{w}\,(-1\,;-9\,;-7)$

Show that the vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ are coplanar.

(2) Consider the three vectors:
$\vec{u}\,(2\,;-2\,;1)$; $\vec{v}\,(1\,;4\,;-2)$; $\vec{w}\,(1\,;-16\,;6)$

Show that the vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ are not coplanar.

E.6947 Consider space with a reference frame.
In each case and without justification, give the relation $\vec{w}=\alpha\cdot\vec{u}+\beta\cdot\vec{v}$ justifying the coplanarity of the vectors $\vec{u}$, $\vec{v}$, $\vec{w}$.

(1) $\vec{u}\,(3\,;2\,;0)$; $\vec{v}\,(1\,;0\,;1)$; $\vec{w}\,(7\,;6\,;-2)$

(2) $\vec{u}\,(1\,;0\,;-1)$; $\vec{v}\,(3\,;-1\,;2)$; $\vec{w}\,(1\,;-1\,;4)$

E.2780 In this exercise, we consider the following two systems of three equations with three unknowns:

$$(S) : \begin{cases} 5a - 2b \;3\; 11c = 0 \\ -a + 3b + 2c = 0 \\ -a + b - c = 0 \end{cases}$$

$$(T) : \begin{cases} -2a + b - 5c = 0 \\ a - 3b - 5c = 0 \\ 5a - 2b + 14c = 0 \end{cases}$$

We place ourselves in a reference frame $(O\,;I\,;J\,;K)$ to study the coplanarity of vectors in space:

(1) (a) Show that the system $\big(S\big)$ admits only the triplet $\big(0\,;\,0\,;\,0\big)$ as a solution.

(b) Deduce that the vectors:
$\vec{p}\,\big(5\,;\,-1\,;\,-1\big)$; $\vec{q}\,\big(-2\,;\,3\,;\,1\big)$; $\vec{s}\,\big(11\,;\,2\,;\,-1\big)$
are non-coplanar.

(2) Consider the following three vectors:

$\vec{u}\,\big(-2\,;\,1\,;\,5\big)$; $\vec{v}\,\big(1\,;\,-3\,;\,-2\big)$; $\vec{w}\,\big(-5\,;-5\,;14\big)$

(a) Justify that the coplanarity of these three vectors is equivalent to the condition:
$$\mathcal{S}_{(T)} \neq \Big\{\big(0\,;\,0\,;\,0\big)\Big\}$$

(b) Deduce that the three vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ are coplanar.

E.2791 The space is given a reference frame $(O\,;I\,;J\,;K)$:

(1) Consider the following four points:

$A(-5\,;-2\,;3)$; $B(0\,;0\,;6)$

$C(-7\,;-1\,;7)$; $D(-21\,;-3\,;9)$

Show that the points A, B, C, D are coplanar.

(2) Consider the following 5 points:

$E(-2\,;1\,;-1)$; $F(-4\,;3\,;-2)$

$G(-3\,;4\,;-4)$; $H(-5\,;6\,;2)$; $L(-11\,;8\,;5)$

Show that the straight line (HL) is not parallel to the plane (EFG).

E.2800 In space with a reference frame $(O\,;I\,;J\,K)$, consider the following four points:

$A(5\,;-4\,;3)$; $B(7\,;-5\,;6)$

$C(10\,;-2\,;1)$; $D(-11\,;-14\,;17)$

Show that the points A, B, C, D are coplanar.

E.2815 Consider the following two vectors defined by their coordinates in a reference frame:

$\vec{u}=(3\,;\,2\,;1)$; $\vec{v}=(-1\,;\,3\,;\,1)$

Consider the vector $\vec{w}$ defined as a function of x a real number by its coordinates:
$\vec{w}\,(2\,;\,16\,;\,x)$

Determine le*(s)* valeur*(s)* of x tel*(les)* that the vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ are coplanar.

E.5437 In space provided with a reference frame $(O\,;I\,;J\,;K)$ orthonormal, consider four points marked by their coordinates:

$A(3\,;-1\,;5)$; $B(-2\,;2\,;3)$; $C(-1\,;-2\,;4)$; $D(5\,;8\,;4)$

Are the points A, B, C, D coplanar?

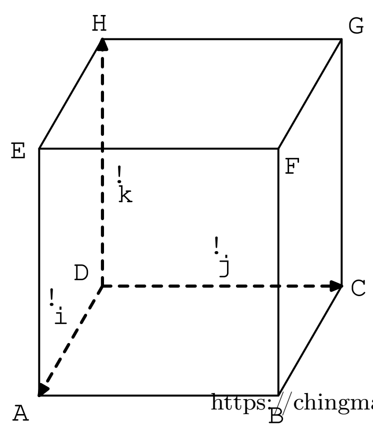
E.6316 In space provided with a reference frame $(O\,;I\,;J\,;K)$ orthonormal, consider four points marked by their coordinates:

$A(3\,;-1\,;5)$; $B(-2\,;2\,;3)$; $C(-1\,;-2\,;4)$; $D(5\,;8\,;4)$

Are the points A, B, C, D coplanar?

8. *Cartesian equation of the plane*

E.4125 Consider the cube $ABCDEFGH$ shown below:



The space is given the reference frame $\left(D\,;\,\overrightarrow{DA}\,;\,\overrightarrow{DC}\,;\,\overrightarrow{DH}\right)$.

1. Name the planes admitting the following standard forms:

   a. $z=0$ b. $y=1$

   c. $x+y=1$ d. $x+y+z=2$

   e. $x+y+z=1$ f. $x-y=0$

2. Determine the standard form of each of the following planes:

   a. (EHD) b. (FGH) c. (HDC)

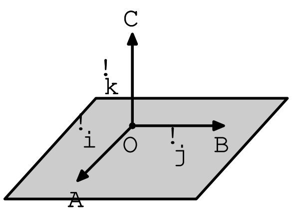
3. a. Justify that the vector $\overrightarrow{BG}$ is orthogonal to the plane (EFC).

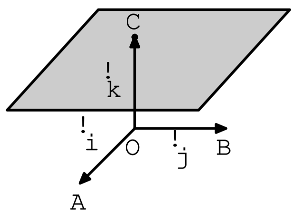
   b. Deduce an equation of the plane (EFC).

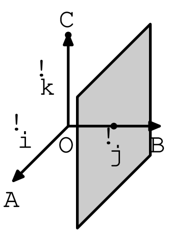
E.4119 Consider the following four points:

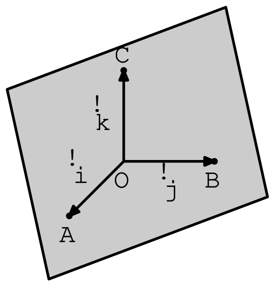
1. the plane $(\mathcal{P}_1)$ is parallel to the plane (OAB) and passes through the point C;
2. plane $(\mathcal{P}_2)$ is pass through the points A, B, C;
3. the $(\mathcal{P}_3)$ median plane of the segment $[OB]$;
4. the plane $(\mathcal{P}_4)$ is parallel to the plane (OAB) and passes through the point O;

Associate each plane with one of the representations below and give its standard form.









E.5418 In the space equipped with a reference point $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$, we consider the plane $(\mathcal{P})$ passing through the point $A(1\,;2\,;-1)$ and admitting the vector $\overrightarrow{n}\,(1\,;-1\,;3)$ as its normal vector.

1. Determine a Cartesian equation for the plane $\mathcal{P}$.
2. Do the points $B\,(2\,;8\,;1)$ and $C(-2\,;5\,;1)$ belong to the plane $\mathcal{P}$?

E.5416 In space provided with a $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$ orthonormal, consider the plane $(\mathcal{P})$ passing through the point A and admitting $\overrightarrow{n}$ as normal vector où:

$$A(3\,;1\,;2) \quad ; \quad \overrightarrow{n}\,(2\,;1\,;-1)$$

Consider the points M and N two points in the space où:

$$M(4\,;-2\,;1) \quad ; \quad N(-2\,;8\,;2)$$

Do the points M and N belong to the $(\mathcal{P})$ plane?

E.4092 The space E is referred to a $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$ orthonormal reference frame. Consider the points A, B and C of coordinates $(1\,;0\,;2)$, $(1\,;1\,;4)$ et $(-1\,;1\,;1)$.

1. Show that the points A, B and C are not aligned.
2. Let $\overrightarrow{n}$ be the coordinate vector $(3\,;4\,;-2)$.

   a. Check that the vector $\overrightarrow{n}$ is orthogonal to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

   b. Deduce a standard form of the plane (ABC).

E.4097 Space is provided with an orthonormal reference frame $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$.
Consider the points:

$$A(1\,;-1\,;4) \quad ; \quad B(7\,;-1\,;-2) \quad ; \quad C(1\,;5\,;-2)$$

1. Justify that the three points A, B, C form a plane.
2. Show that the vector $\overrightarrow{n}\,(1\,;1\,;1)$ is a normal vector to the plane (ABC).
3. Deduce that $x+y+z-4=0$ is a standard form of the plane (ABC).

E.4095 Space is referred to a $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$ orthonormal reference frame. Consider the two points:

$$A\,(8\,;0\,;8) \quad ; \quad B\,(10\,;3\,;10)$$

as well as the straight line (d) admitting as parametric representation:

$$(d):\ \begin{cases} x = -5+3t \\ y = 1+2t \\ z = -2t \end{cases} \quad \text{où } t\in\mathbb{R}$$

1. a. Give a parametric representation of the line (Δ) defined by A and B.

   b. Demonstrate that (d) and (Δ) are non-coplanar.

2. The $(\mathcal{P})$ plane is parallel to (d) and contains (Δ). Show that the vector $\overrightarrow{n}\,(2\,;-2\,;1)$ is a normal vector to $(\mathcal{P})$. Determine a standard form of the plane $(\mathcal{P})$.

9. *Cartesian equation of the plane - finding the normal vector*

E.4111 Space is provided with an orthonormal reference frame $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$.

Consider the plane $(\mathcal{P})$ admitting the following standard form:

$$(\mathcal{P}):\ 5{\cdot}x-2{\cdot}y+z-5=0$$

1. Give the coordinates of a vector $\overrightarrow{n}$ normal to the plane $(\mathcal{P})$.
2. Determine the equation of the plane $(\mathcal{Q})$ parallel to the

plane $(\mathcal{P})$ and passing through the point $A(5\,;-1\,;2)$

E.5419 In space with a reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$, consider the three points:

$A(1\,;1\,;-2)$; $B(3\,;-1\,;2)$; $C(0\,;2\,;1)$

1. Justify that the three points define a plane.
2. Determine a non-zero normal vector to the plane (ABC) having its integer coordinates.
3. Determine a standard form of the plane (ABC).

E.4112 Space is provided with an orthonormal reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$.

Consider the points $A(2\,;-3\,;-1)$ and $B(-1\,;1\,;0)$.

Determine the equation of the mediator plane of segment $[AB]$.

E.4251 In the plane provided with an orthonormal reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$, we give the three points:

$A(1\,;2\,;-1)$; $B(-3\,;-2\,;3)$; $C(0\,;-2\,;-3)$

1. Demonstrate that the vector $\vec{n}(2\,;-1\,;1)$ is a normal vector to the plane (ABC).
2. Determine a standard form of the plane (ABC).

E.4245 Space is provided with an orthonormal reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$.

we consider the points:
$A(1\,;-1\,;4)$; $B(7\,;-1\,;-2)$; $C(1\,;5\,;-2)$

1. Calculate the coordinates of the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$.
2. Show that the triangle ABC is equilateral.
3. Show that the vector $\vec{n}(1\,;1\,;1)$ is a normal vector to the plane (ABC).
4. Deduce that $x+y+z-4=0$ is a standard form of the plane (ABC).

10. Relative positions of planes

E.5420 In space provided with a reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$, consider the two planes $(\mathcal{P})$ and $(\mathcal{P}')$ admitting as standard form:

$(\mathcal{P}):\ 2x-y+z+3=0$; $(\mathcal{P}'):\ -4x+2y-2z-1=0$

Justify that these two planes are parallel.

E.5463 In space provided with an orthonormal reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$, we give the three points:

$A(1\,;2\,;-1)$; $B(-3\,;-2\,;3)$; $C(0\,;-2\,;-3)$

1. a. Demonstrate that the points A, B and C are not aligned.
   b. Demonstrate that the vector $\vec{n}(2\,;-1\,;1)$ is a normal vector to the plane (ABC).
2. Let $(\mathcal{P})$ be the plane whose standard form is:
   $x+y-z+2=0$
   Show that the planes (ABC) and $(\mathcal{P})$ are perpendicular.

E.5536 Consider:
- (P_1) the plane of equation: $x+y+z=0$;
- (P_2) the plane of equation: $x+4y+2=0$.

1. Show that the planes (P_1) and (P_2) intersect.
2. Verify that the straight line (d), intersection of the planes (P_1) and (P_2) has as parametric representation:

$$\begin{cases} x = -4t - 2 \\ y = t \\ z = 3t + 2 \end{cases} \quad \text{où } t\in\mathbb{R}.$$

11. Secant plane and parametric representation of intersection

E.3129 Space is provided with an orthonormal reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$.
Let $(\mathcal{P}_1)$ be the plane with Cartesian equation $-2x+y+z-6=0$ and $(\mathcal{P}_2)$ be the plane with Cartesian equation $x-2y+4z-9=0$.

1. Show that $(\mathcal{P}_1)$ and $(\mathcal{P}_2)$ are perpendicular.

   Recall that two planes are perpendicular if, and only if, a non-zero normal vector to one is orthogonal to a non-zero normal vector to the other.
2. Let (D) be the line of intersection of $(\mathcal{P}_1)$ and $(\mathcal{P}_2)$.
   Show that a parametric representation of (d) is:

$$\begin{cases} x = -7 + 2t \\ y = -8 + 3t \\ z = t \end{cases}$$

E.5421 In space provided with a reference frame $(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k})$, consider the two planes $(\mathcal{P})$ and $(\mathcal{P}')$ admitting as standard form:

$(\mathcal{P}):\ x+2y-z+3=0$; $(\mathcal{P}'):\ 4x-2y+z-1=0$

1. Justify that these two planes are secant.
2. Determine a parametric representation of the line (d) intersection of the planes $(\mathcal{P})$ and $(\mathcal{P}')$.

E.4093 In space provided with a $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$ orthonomal, consider the two planes $(\mathcal{P}_1)$ and $(\mathcal{P}_2)$ admitting as standard forms:

$$(\mathcal{P}_1):\ 2x+y+2z+1=0 \quad ; \quad (\mathcal{P}_2):\ x-2y+6z=0$$

Show that the planes $(\mathcal{P}_1)$ and $(\mathcal{P}_2)$ intersect along a straight line (d), a parametric representation of which will be determined.

E.4039 Space is referred to an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$. Consider:

- the four points:
  $A\,(0\,;0\,;3)$; $E\,(2\,;0\,;4)$; $C\,(-1\,;1\,;2)$; $D(1\,;-4\,;0)$
- both shots:
  $(\mathcal{P}_1):\ 7x+4y-3z+9=0$; $(\mathcal{P}_2):\ x-2y=0$.
- The two straight lines admitting as parametric representation:

$$(\Delta_1)\begin{cases} x = -1+t \\ y = -8+2t \\ z = -10+5t \end{cases} \text{où } t\in\mathbb{R} \quad ; \quad (\Delta_2)\begin{cases} x = 7+2t' \\ y = 8+4t' \\ z = 8-t' \end{cases} \text{où } t'\in\mathbb{R}.$$

For each question, only one of the four propositions is correct. The candidate will indicate on the copy the number of the question and the letter corresponding to the chosen answer. No justification is required.

A correct answer earns 0.5 point; an incorrect answer deducts 0.25 point; the absence of an answer is counted 0 point. If the total is negative, the mark is reduced to 0.

Le plan $(\mathcal{P}_1)$ est	Le plan (ABC)	Le plan (BCD)	Le plan (ACD)	Le plan (AED)
The straight line (Δ_1) contient	The point A	The point B	The point C	The point D
Relative position of $(\mathcal{P}_1)$ and de (Δ_1)	(Δ_1) est strictement parallèle à $(\mathcal{P}_1)$	(Δ_1) est incluse dans $(\mathcal{P}_1)$	(Δ_1) coupe $(\mathcal{P}_1)$	(Δ_1) est orthogonale à $(\mathcal{P}_1)$
Position relative de (Δ_1) et de (Δ_2)	(Δ_1) est strictement parallèle à (Δ_2)	(Δ_1) et (Δ_2) sont confondues	(Δ_1) et (Δ_2) sont sécantes	(Δ_1) et (Δ_2) sont non coplanaires.
L'intersection of $(\mathcal{P}_1)$ et of $(\mathcal{P}_2)$ est une droite dont une représentation paramétrique est	$\begin{cases} x= t \\ y=-2+\frac{1}{2}t \\ z= 3t \end{cases}$	$\begin{cases} x= 2t \\ y= t \\ z=3+6t \end{cases}$	$\begin{cases} x= 5t \\ y=1-2t \\ z= t \end{cases}$	$\begin{cases} x=-1+t \\ y=2+t \\ z=-3t \end{cases}$

E.5537 Consider the two planes (P_1) and (P_2) with standard forms:

$$(P_1):\ 2x+y-z+1=0 \quad ; \quad (P_2):\ x-y+2z+3=0$$

1. Justify that the planes (P_1) and (P_2) are secant.
2. Determine a parametric equation of the line of intersection of the planes (P_1) and (P_2).

E.4131 Space is referred to the orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$.

Consider the planes $(\mathcal{P})$ and $(\mathcal{Q})$ of equations:
$x+y+z=0$; $2x+3y+z-4=0$

1. Show that the instersection of the planes $(\mathcal{P})$ and $(\mathcal{Q})$ is the straight line (d) whose parametric representation is:
$$\begin{cases} x = -4-2t \\ y = 4+t \\ z = t \end{cases} \text{où } t\in\mathbb{R}.$$
2. Let λ be a real number.
   Consider the plane $(\mathcal{P}_\lambda)$ of equation:
   $(1-\lambda)\cdot(x+y+z)+\lambda\cdot(2x+3y+z-4)=0$
   a. Verify that the vector $\vec{n}\,(1+\lambda\,;1+2\lambda\,;1)$ is a normal vector to the $(\mathcal{P}_\lambda)$ plane.
   b. Give a value of the real number λ for which the planes $(\mathcal{P})$ and $(\mathcal{P}_\lambda)$ are coincident.
   c. Is there a real number λ for which the planes $(\mathcal{P})$ and $(\mathcal{P}_\lambda)$ are perpendicular?
3. Determine a parametric representation of the line (d'), intersection of the planes $(\mathcal{P})$ and $(\mathcal{P}_{-1})$.
   Show that the straight lines (d) and (d') are coincident.
4. *In this question, any trace of research, however incomplete, or initiative, however unsuccessful, will be taken into account in the assessment.*

   Consider the point $A\,(1\,;1\,;1)$.

   Determine the distance from the point A to the line (d), i.e. the distance between the point A and its orthogonal projected onto the line (d).

12. *Relative positions of lines and planes*

E.4088 Space is provided with an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$.

We consider:

- $(\mathcal{P})$ is the plane passing through $A(3\,;1\,;2)$ and of normal vector $\vec{n}\,(1\,;-4\,;1)$;
- (d) is the straight line passing through $B(1\,;4\,;2)$ with director vector $\vec{u}\,(1\,;1\,;3)$.

1. Show that the plane $(\mathcal{P})$ has standard form:
   $x-4y+z-1=0$
2. Show that the line (d) is strictly parallel to the plane $\mathcal{P}$.

E.5451 In space referred to an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$, consider the straight line (d), a parametric representation of which is given, and the plane $(\mathcal{P})$, a standard form of which is given:

$$(d):\ \begin{cases} x = 1-2t \\ y = t \\ z = -5-4t \end{cases} \text{ où } t\in\mathbb{R} \quad ; \quad (\mathcal{P}):\ 3x+2y-z-5=0$$

Show that the line (d) is strictly parallel to the plane $(\mathcal{P})$.

E.5422 In space provided with a reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$, we consider the straight line (d) admitting the parametric representation and the plane $(\mathcal{P})$ admitting the standard form defined by:

$$(d):\ \begin{cases} x = 3-t \\ y = 2-t \\ z = t \end{cases} \quad ; \quad (\mathcal{P}):\ 2x-4y-2z+3=0$$

1. a. Justify that the line (d) is parallel to the plane $(\mathcal{P})$.
   b. Is the line (d) included in the plane $(\mathcal{P})$?
2. Consider the plane $(\mathcal{P}')$ admitting as standard form:
   $(\mathcal{P}'):\ 2x-4y-2z+c=0$ où $c\in\mathbb{R}$

   Determine the value of parameter c so that the straight line (d) is included in the plane $(\mathcal{P}')$.

E.4083 Space is referred to a $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$ direct orthonormal reference frame. Consider the points:

$$A(-2\,;0\,;1) \quad ; \quad B(1\,;2\,;-1) \quad ; \quad C(-2\,;2\,;2)$$

We'll admit that the points A, B and C are not aligned.

1. Verify that a standard form of the plane (ABC) is:
   $2x-y+2z+2=0$
2. Let $(\mathcal{P}_1)$ and $(\mathcal{P}_2)$ be the planes of equations:
   $x+y-3z+3=0 \quad ; \quad x-2y+6z=0$

   Show that the planes $(\mathcal{P}_1)$ and $(\mathcal{P}_2)$ intersect along a straight line (d) of which a system of parametric equations is:

$$\begin{cases} x = -2 \\ y = -1+3t \\ z = t \end{cases} \text{ où } t\in\mathbb{R}$$

3. Show that the straight line (d) and the plane (ABC) are secant and determine the coordinates of their point of intersection.

E.5423 In space provided with a reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$, consider the straight line (d) admitting the parametric representation and the plane $(\mathcal{P})$ admitting the standard form defined by:

$$(d):\ \begin{cases} x = 1+t \\ y = 2-t \\ z = 3+2t \end{cases} \quad ; \quad (\mathcal{P}):\ 3x-y+2z+1=0$$

1. Justify that the straight line (d) is secant to the plane $(\mathcal{P})$.
2. Determine the coordinates of the point of intersection of the line (d) and the plane $(\mathcal{P})$.

E.3113 Consider the parametric line:

$$\begin{cases} x = t+2 \\ y = -2t \\ z = 3t-1 \end{cases} \text{ where } t\in\mathbb{R}$$

and the plane whose Cartesian equation is:
$x+2\cdot y+z-3=0$

Justify that this plane and line are parallel.

E.4037 Space is provided with an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$.

Note (D) the straight line passing through the points $A(1\,;-2\,;-1)$ and $B(3\,;-5\,;-2)$.

1. Show that a parametric representation of the line (D) is:

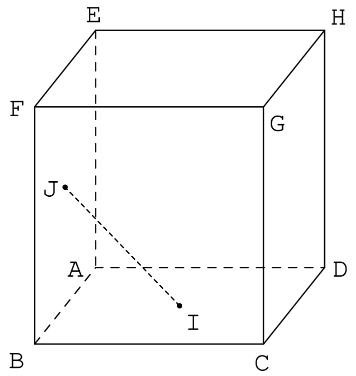
$$\begin{cases} x = 1+2t \\ y = -2-3t \\ z = -1-t \end{cases} \text{ with } t\in\mathbb{R}.$$

2. Note (D') the straight line whose parametric representation is:

$$\begin{cases} x = 2-k \\ y = 1+2k \\ z = k \end{cases} \text{ with } k\in\mathbb{R}.$$

   Show that the straight lines (D) and (D') are not coplanar.
3. Consider the plane $(\mathcal{P})$ of equation $4\cdot x+y+5\cdot z+3=0$.
   a. Show that the plane $(\mathcal{P})$ contains the straight line (D).
   b. Show that the plane $(\mathcal{P})$ and the straight line (D') intersect at a point C whose coordinates will be specified.

E.4129 In space, consider the cube $ABCDEFGH$. The points I and J represent the centers of the faces $ABCD$ and $ABFE$, respectively.



The space is given the reference frame $\left(A\,;\,\overrightarrow{AB}\,;\,\overrightarrow{AD}\,;\,\overrightarrow{AE}\right)$.

1. a. Give the coordinates of points I and J.
   b. Give a parametric representation of the line (IJ).
2. Consider the straight line (Δ) admitting the parametric representation:
$$\begin{cases} x = 1 + t \\ y = \dfrac{1}{2} + 2\cdot t \\ z = -t \end{cases} \quad \text{où } t\in\mathbb{R}$$
Show that the straight lines (Δ) and (IJ) are non-coplanar.
3. a. Justify that the plane (AGH) admits the equation: $y - z = 0$
   b. Determine the coordinates of the point of intersection of the line (IJ) and the plane (AGH).

E.5450 In the orthonormal reference frame $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$ of space, consider the planes $(\mathcal{P})$ and $(\mathcal{P}')$ of equations:
$$(\mathcal{P}):\ x - y - z - 2 = 0 \quad ; \quad (\mathcal{P}'):\ x + y + 3z = 0$$

1. Consider the straight line (d) admitting as parametric representation:
$$\begin{cases} x = -3 - 2t \\ y = 2t \\ z = 1 + 2t \end{cases} \quad \text{où } t\in\mathbb{R}$$
   a. Justify that the line (d) is orthogonal to the plane $(\mathcal{P})$.
   b. Determine the coordinates of the point M of intersection of the plane $(\mathcal{P})$ with the line (d).
2. Consider the straight line (Δ) with parametric representation:
$$\begin{cases} x = 1 - t' \\ y = -1 - 2t' \\ z = t' \end{cases} \quad \text{où } t'\in\mathbb{R}$$
   a. Justify that the planes $(\mathcal{P})$ and $(\mathcal{P}')$ are secant.
   b. Show that the line (Δ) is the line of intersection of the planes $(\mathcal{P})$ and $(\mathcal{P}')$.

13. *Cartesian equation of the plane and orthogonal projection*

E.3213 *For this exercise, copy for each question, your answer.*
Each correct answer earns 1 point. No answer is penalized.
0.5 points will be deducted for each wrong answer.
The final mark for the exercise may not be lower than zero.

Soit $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$ un repère orthonormé.

1. The line passing through $A\left(1\,;2\,;-4\right)$ and $B\left(-3\,;4\,;1\right)$ and the straight line (d) represented by:
$$\begin{cases} x = -11 - 4t \\ y = 8 + 2t \\ z = 11 + 5t \end{cases} \quad \text{où } t \in \mathbb{R}$$
   a. sécantes
   b. strictement parallèles
   c. confondues
   d. non coplanaires
2. Let $\mathscr{P}$ be the plane with equation $2x+3y-z+4=0$ and the line $\mathscr{D}$ represented by:
$$\begin{cases} x = t \\ y = t \\ z = 8 + t \end{cases} \quad \text{où } t\in\mathbb{R}$$
   a. $\mathscr{P}$ et $\mathscr{D}$ sont sécants
   b. $\mathscr{P}$ et $\mathscr{D}$ sont strictement parallèles
   c. $\mathscr{D}$ is included in $\mathscr{P}$
   d. None of these possibilities is true.
3. The distance of the point $A\left(1\,;2\,;-4\right)$ from the plane of equation $2x + 3y - z + 4 = 0$ is:
   a. $\dfrac{8\sqrt{14}}{7}$ b. 16 c. $8\sqrt{14}$ d. $\dfrac{8}{7}$
4. Let be the point $B\left(-3\,;4\,;1\right)$ and the sphere $\mathscr{S}$ of equation $x^2 + y^2 + z^2 = 16$:
   a. B is inside $\mathscr{S}$
   b. B is outside $\mathscr{S}$
   c. B is on $\mathscr{S}$
   d. We don't know pas

E.4242 In space provided with an orthonormal reference frame $\left(O\,;\,\overrightarrow{i}\,;\,\overrightarrow{j}\,;\,\overrightarrow{k}\right)$, we give the points:
$$A(3\,;2\,;-1) \quad ; \quad B(-6\,;1\,;1)$$
$$C(4\,;-3\,;3) \quad ; \quad D(-1\,;-5\,;-1)$$

1. Verify that a standard form of the (BCD) plane is: $-2\cdot x - 3\cdot y + 4\cdot z - 13 = 0$
2. Determine the coordinates of the point H, orthogonal projected from the point A on the plane (BCD).
3. Calculate the scalar product: $\overrightarrow{BH}\cdot\overrightarrow{CD}$

E.4240 Space is referred to an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$.

Consider the plane $\mathcal{P}$ of equation $-x+3\cdot y-z+5=0$ and the point $A\left(1\,;-2\,;1\right)$

1. Determine the parametric representation of the line (d) passing through the point A and orthogonal to the plane $\mathcal{P}$.
2. Deduce the coordinates of the point H projected orthogonally from the point A onto the plane $\mathcal{P}$.
3. Determine the distance from point A to plane $\mathcal{P}$.

E.6084 We place ourselves in space provided with an orthonormal reference point.

Consider the plane $\mathcal{P}$ of equation:

$$x-y+3z+1=0$$

and the straight line $\mathcal{D}$ whose parametric representation is:

$$\begin{cases} x = 2t \\ y = 1 + t \\ z = -5 + 3t \end{cases} \quad , \quad \text{où } t\in\mathbb{R}$$

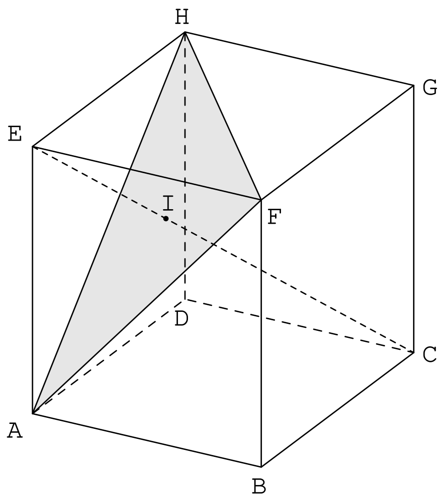
We give the points:

$$A\left(1\,;1\,;0\right) \quad ; \quad B\left(3\,;0\,;-1\right) \quad ; \quad C\left(7\,;1\,;-2\right)$$

1. Justify that the line $\mathcal{D}$ and the plane $\mathcal{P}$ are secant. Determine the coordinates of their point of intersection.
2. Are the planes $\mathcal{P}$ and (ABC) parallel? Justify your answer?
3. Determine the coordinates of point M projected orthogonally from point A onto the plane $\mathcal{P}$.

14. *A little more*

E.4317 *Consider a cube $ABCDEFGH$, with edge length 1. Note I the point of intersection of the line (EC) and the plane (AFH).*



1. We place ourselves in the reference frame $\left(D\,;\,\overrightarrow{DA}\,;\,\overrightarrow{DC}\,;\,\overrightarrow{DH}\right)$. In this reference frame, the cube's vertices have the coordinates:

$$A\left(1\,;0\,;0\right) \quad ; \quad B\left(1\,;1\,;0\right) \quad ; \quad C\left(0\,;1\,;0\right) \quad ; \quad D\left(0\,;0\,;0\right)$$

$$E\left(1\,;0\,;1\right) \quad ; \quad F\left(1\,;1\,;1\right) \quad ; \quad G\left(0\,;1\,;1\right) \quad ; \quad H\left(0\,;0\,;1\right)$$

   a. Determine a parametric representation of the line (EC).
   b. Determine a standard form of the plane (AFH).
   c. Deduce the coordinates of the point I, then show that the point I is the orthogonal project of the point E onto the plane (AFH).
   d. Verify that the distance from point E to plane (AFH) is equal to $\dfrac{\sqrt{3}}{3}$.
   e. Show that the line (HI) is perpendicular to the line (AF).
   What does the point I represent for the triangle AFH?

2. *In the remainder of this exercise, any trace of research, however incomplete, or initiative, however unsuccessful, will be taken into account in the assessment.*

   Definitions:
   - A tetrahedron is said to be of type 1 if its faces have the same area;
   - it is said to be of type 2 if the opposite edges are orthogonal two by two;
   - it is said to be of type 3 if it is both of type 1 and of type 2.

   Specify from quel*(s)* type*(s)* is the tátrahedron $EAFH$.

E.5462 Space is referred to an orthonormal reference frame $\left(O\,;\,\vec{i}\,;\,\vec{j}\,;\,\vec{k}\right)$.

Let $(\mathcal{P})$ be the plane with equation $3x+y-z-1=0$ and (d) be the line whose parametric representation is:

$$\begin{cases} x = -t+1 \\ y = 2t \\ z = -t+2 \end{cases} \quad \text{où } t \text{ denotes a real number.}$$

1. a. Does the point $C\left(1\,;3\,;2\right)$ belong to the plane $(\mathcal{P})$? Justify.
   b. Demonstrate that the line (d) is included in the plane $(\mathcal{P})$.
2. Let $(\mathcal{Q})$ be the plane passing through the point C and orthogonal to the line (d).
   a. Determine a standard form of the plane $(\mathcal{Q})$.
   b. Calculate the coordinates of point I, point of intersection of plane $(\mathcal{Q})$ and line (d).
   c. Show that: $CI=\sqrt{3}$.
3. Let t be a real number and M_t be the point on the line (d) with coordinates: $M_t\left(-t+1\,;2t\,;-t+2\right)$
   a. Verify that for any real number t,:
   $$C{M_t}^2 = 6t^2-12t+9.$$
   b. Show that CI is the minimum value of CM_t when t describes the set of real numbers.

15. Course - Vectors and spaces

E.5504 Consider space with a reference frame $(O\,;\vec{i}\,;\vec{j}\,;\vec{k})$ orthonormal:

(1) $(\mathcal{P})$ a plane passing through the point A and admitting the vector $\vec{n}\,(a\,;b\,;c)$ as normal vector.

Show that the plane $(\mathcal{P})$ admits as standard form the following equation:
$$a{\cdot}x+b{\cdot}y+c{\cdot}z+d=0 \quad \text{où } d\in\mathbb{R}.$$

(2) Let a, b, c, d be four real numbers où a, b, c are all non-zero. Show that the set of points verifying the standard form below is a plane of space:
$$a{\cdot}x+b{\cdot}y+c{\cdot}z+d=0$$

E.5505 A line is orthogonal to a plane if, and only if, it is orthogonal to two intersecting lines in that plane.

16. Old annuals (before 2012)

E.3153 Space is provided with an orthonormal reference frame $\left(O\,;\vec{i}\,;\vec{j}\,;\vec{k}\right)$.

(1) Consider the plane $\mathcal{P}$ passing through the point $B\big(1\,;-2\,;1\big)$ and of normal vector $\vec{n}\,(-2\,;1\,;5)$ and the plane $\mathcal{R}$ with Cartesian equation $x+2y-7=0$

(a) Demonstrate that the planes $\mathcal{P}$ and $\mathcal{R}$ are perpendicular.

(b) Demonstrate that the intersection of the planes $\mathcal{P}$ and $\mathcal{R}$ is the line Δ passing through the point $C\big(-1\,;4\,;-1\big)$ and direction vector $\vec{u}\,(2\,;-1\,;1)$.

(c) Let the point $A\big(5\,;-2\,;-1\big)$. Calculate the distance from point A to plane $\mathcal{P}$ and then the distance from point A to plane $\mathcal{R}$.

(d) Determine the distance from point A to the line Δ.

(2) (a) Let, for any real number t, the point M_t of coordinates $\big(1+2{\cdot}t\,;3-t\,;t\big)$.
Determine as a function of t the length AM_t.

We denote $\varphi(t)$ this length. We thus define a function φ of $\mathbb{R}$ in $\mathbb{R}$.

(a) Study the direction of variation of the function φ on $\mathbb{R}$; specify its minimum.

(b) Interpret the value of this minimum geometrically.

E.4055 Space is provided with an orthonormal reference frame $(O\,;\vec{i}\,;\vec{j}\,;\vec{k})$.

The aim of this exercise is to determine the relative position of objects in space.

$\mathcal{P}$ is the plane passing through $A\,\big(3\,;1\,;2\big)$ and of normal vector $\vec{n}\,\big(1\,;-4\,;1\big)$;

$\mathcal{D}$ is the straight line passing through $B\,\big(1\,;4\,;2\big)$ de vecteur directeur $\vec{u}\,\big(1\,;1\,;3\big)$.

$\mathcal{S}$ is the sphere with center $\Omega\,\big(1\,;9\,;0\big)$ passing through A.

(1) **Intersection of the plane $\mathcal{P}$ and the straight line $\mathcal{D}$.**

(a) Demonstrate that the plane $\mathcal{P}$ has standard form:
$$x-4y+z-1=0$$

(b) Show that the line $\mathcal{D}$ is strictly parallel to the plane $\mathcal{P}$.

(2) **Intersection of the plane $\mathcal{P}$ and the sphere $\mathcal{S}$.**

(a) Calculate the distance d from the point Ω to the plane $\mathcal{P}$.

(b) Calculate the radius of the sphere $\mathcal{S}$. Deduce the intersection of the plane $\mathcal{P}$ and the sphere $\mathcal{S}$.

(3) **Intersection of the line $\mathcal{D}$ and the sphere $\mathcal{S}$.**

(a) Determine a parametric representation of the line $\mathcal{D}$.

(b) Determine a standard form of the sphere $\mathcal{S}$.

(c) Deduce that the line $\mathcal{D}$ intersects the sphere $\mathcal{S}$ at two distinct points M and N whose coordinates we will not attempt to determine.

E.3126 Space is provided with a $\left(O\,;\vec{i}\,;\vec{j}\,;\vec{k}\right)$ direct orthonormal coordinate system.

No figures are required.

Questions (3) and (4) are independent of questions (1) et (2).

Consider the four points A, B, C and I with coordinates:
$$A\begin{pmatrix}-1\\2\\1\end{pmatrix}\;;\;B\begin{pmatrix}1\\-6\\-1\end{pmatrix}\;;\;C\begin{pmatrix}2\\2\\2\end{pmatrix}\;;\;I\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

(1) (a) Calculate the vector product $\overrightarrow{AB}\wedge\overrightarrow{AC}$.

(b) Determine a Cartesian equation of the plane containing the three points A, b, C.

(2) Let (Q) be the plane of equation: $x+y-3z+2=0$ and (Q') the reference plane $\left(O\,;\vec{i}\,;\vec{j}\,;\vec{k}\right)$.

(a) Why are (Q) and (Q') secant?

(b) Give a point E and a directing vector $\vec{u}$ of the line of intersection (Δ) of the planes (Q) and (Q').

(3) Write a Cartesian equation of the sphere S of center I and radius 2.

(4) Consider the points J and K with coordinates:
$$\begin{pmatrix}-2\\0\\0\end{pmatrix}\;;\;\begin{pmatrix}1\\0\\1\end{pmatrix}$$

Carefully determine the intersection of the sphere (S) and the straight line (JK).

17. Course - old program: Spaces

E.3906 Let D be the point with coordinates $(x_D\,;\,y_D\,;\,z_D)$ and $\mathcal{P}$ the equation plane $a{\cdot}x+b{\cdot}y+c{\cdot}z+d=0$, où a, b and c are real numbers that are not all zero.

Show that the distance from point D to plane P is given by:

$$d(D,\mathcal{P}) = \frac{|a{\cdot}x_D + b{\cdot}y_D + c{\cdot}z_D + d|}{\sqrt{a^2+b^2+c^2}}$$

E.4090 Let a, b, c and d be real numbers such that: $(a\,;b\,;c)\neq(0\,;0\,;0)$.
Let $\mathscr{P}$ be the plane of equation $a{\cdot}x+b{\cdot}y+c{\cdot}z+d=0$.

Consider the point I of coordinates $(x_I\,;y_I\,;z_I)$ and the vector $\overrightarrow{n}$ de coordonnées $(a\,;b\,;c)$.

The aim of this part is to demonstrate that the distance from I to the plane $\mathscr{P}$ is equal to:

$$\frac{|ax_I+by_I+cz_I+d|}{\sqrt{a^2+b^2+c^2}}$$

1. Let Δ be the straight line passing through I and orthogonal to the plane $\mathscr{P}$.
Determine, as a function of a, b, c, x_I, y_I and z_I, a system of parametric equations of Δ.
2. Note H the point of intersection of Δ and $\mathscr{P}$.
   a. Justify that there exists a real k such that: $\overrightarrow{IH}=k{\cdot}\overrightarrow{n}$.
   b. Determine the expression of k as a function of a, b, c, d, x_I, y_I and z_I.
   c. Deduce that: $IH=\dfrac{|a{\cdot}x_I+b{\cdot}y_I+c{\cdot}z_I|}{\sqrt{a^2+b^2+c^2}}$

18. Unclassified financial years

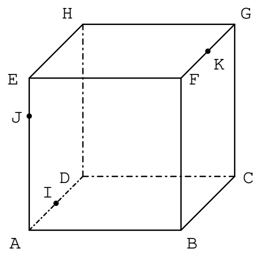
E.7247 Solve the system of equations:

$$\begin{cases} x^2+y^2+z^2=1 \\ x+y+z=1 \end{cases}$$

E.8139

The adjacent figure shows a $ABCDEFGH$ cube. The three points I, J, K are defined by the following conditions:

- I is the middle of the segment $[AD]$
- J is such that: $\overrightarrow{AJ}=\dfrac{3}{4}{\cdot}\overrightarrow{AE}$
- K is the middle of segment $[FG]$.



Let R be the orthogonal project of the point F onto the plane (IJK). The point R is therefore the only point on the plane (IJK) such that the straight line (FR) is orthogonal to the plane (IJK).

We define the interior of the cube as the set of points $M(x\,;y\,;z)$ such that: $\begin{cases} 0<x<1 \\ 0<y<1 \\ 0<z<1 \end{cases}$

Is the point R inside the cube?

E.5417 In space provided with a $(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k})$, consider the plane $(\mathcal{P})$ admitting as standard form:

$$2{\cdot}x+3{\cdot}y-z+1=0$$

Does the vector $\overrightarrow{u}(4\,;-2\,;2)$ admit a representative included in the plane $(\mathcal{P})$.

E.3858 Space is provided with an orthonormal reference frame $(O\,;\overrightarrow{i}\,;\overrightarrow{j}\,;\overrightarrow{k})$.

We consider:

- the plane $\mathcal{P}$ passing through the point $B(1\,;-2\,;1)$ and normal vector $\overrightarrow{n}(-2\,;1\,;5)$;
- the plane $\mathcal{R}$ of standard form: $x+2y-7=0$.

1. Demonstrate that the planes $\mathcal{P}$ and $\mathcal{R}$ are perpendicular.
2. Show that the intersection of the planes $\mathcal{P}$ and $\mathcal{R}$ is the line Δ passing through the point $C(-1\,;4\,;-1)$ and direction vector $\overrightarrow{u}(2\,;-1\,;1)$.
3. Let the point $A(5\,;-2\,;-1)$. Calculate the distance from point A to plane $\mathcal{P}$ and then the distance from point A to plane $\mathcal{R}$.
4. Determine the distance of point A to the line Δ.

E.4089 The plane $\mathscr{Q}$ of equation $x-y+z-11=0$ is tangent to a sphere $\mathscr{S}$ of center the point Ω of coordinates $(1\,;-1\,;3)$.

1. Determine the radius of the sphere $\mathscr{S}$.
2. Determine a system of parametric equations of the line Δ passing through Ω and orthogonal to the plane $\mathscr{Q}$.
3. Deduce the coordinates of the point of intersection of the sphere $\mathscr{S}$ and the plane $\mathscr{Q}$.

E.4106 In space provided with an orthonormal reference frame $(O\,;\vec{i}\,;\vec{j}\,;\vec{k})$, we give the points:

$A(3\,;2\,;-1)$; $B(-6\,;1\,;1)$

$C(4\,;-3\,;3)$; $D(-1\,;-5\,;-1)$

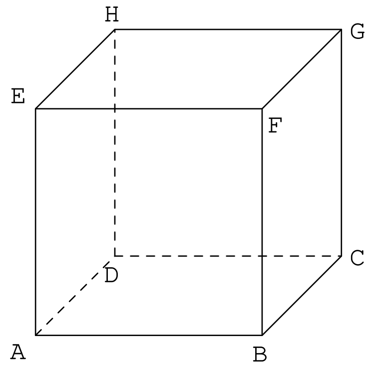
1. Verify that a standard form of the (BCD) plane is:
   $-2{\cdot}x - 3{\cdot}y + 4{\cdot}z - 13 = 0$
2. Determine the coordinates of the point H, orthogonal projected from the point A on the plane (BCD).
3. Calculate the scalar product $\overrightarrow{BH}\cdot\overrightarrow{CD}$.

E.4134 The space is equipped with the orthonormal basis $(O\,;\vec{i}\,;\vec{j}\,;\vec{k})$ and we denote by $\mathcal{P}$ the plane of equation:

$3{\cdot}x + 2{\cdot}y = 29$

1. Prove that $\mathcal{P}$ is parallel to the axis (Oz) with direction vector $\vec{k}$.
2. Determine the coordinates of the points of intersection of the plane $\mathcal{P}$ with the axes (Ox) and (Oy) of the respective direction vectors $\vec{i}$ and $\vec{j}$.
3. Draw a figure and trace the lines of intersection of the plane $\mathcal{P}$ with the three coordinate planes.
4. On the previous figure, place the points whose coordinates are both integer and positive on the line of intersection of planes $\mathcal{P}$ and (xOy).

E.4329 Consider the cube $ABCDEFGH$ with side 1 shown below:



Throughout the exercise, space is referred to the orthonormal reference frame $\left(D\,;\overrightarrow{DA}\,;\overrightarrow{DC}\,;\overrightarrow{DH}\right)$.

Let K be the barycenter of the weighted points $(D\,;1)$ and $(f\,;2)$

Part A

1. Show that the point K has coordinates $\left(\frac{2}{3}\,;\frac{2}{3}\,;\frac{2}{3}\right)$.
2. Show that the straight lines (EK) and (DF) are orthogonal.
3. Calculate the distance EK.

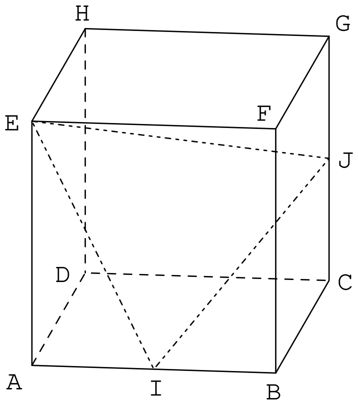
Part B

Let M be a point on segment $[HG]$.
Note $m = HM$ (*m is therefore a real belonging to* $[0\,;1]$).

1. Show that, for any real m belonging to the interval $[0\,;1]$, the volume of the tetrahedron $EMFD$, in units of volume, is equal to $\frac{1}{6}$.
2. Show that a standard form of the plane (MFD) is:
   $(-1+m){\cdot}x + y - m{\cdot}z = 0$.
3. Note d_m the distance of the point E from the plane (MFD).
   a. Show that, for any real m belonging to the interval $[0\,;1]$:
   $$d_m = \frac{1}{\sqrt{2{\cdot}m^2 - 2{\cdot}m + 2}}$$
   b. Determine the position of M on the segment $[HG]$ for which the distance d_m is maximum.
   c. Deduce that when the distance d_m is maximum, the point K is the orthogonal projected of E onto the plane (MFD)

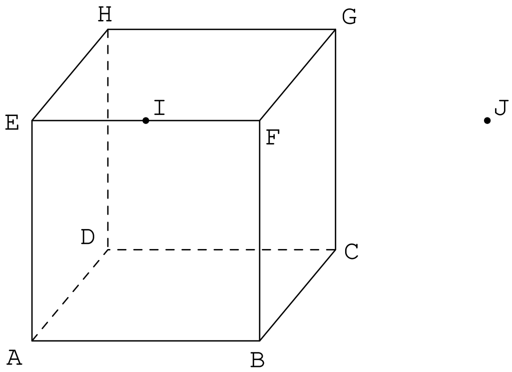
E.6068 Consider the cube $ABCDEFGH$, with edge length 1,

and note I and J the middles of the $[AB]$ and $[CG]$ edges. On utilisera le repère $\left(A\,;\overrightarrow{AB}\,;\overrightarrow{AD}\,;\overrightarrow{AE}\right)$



1. Determine the standard form of the plane EFJ.
2. Determine the coordinates of point P projected from point I onto plane (EFJ).
3. Determine distance IP.
4. Show that the volume of the tetrahedron $EFIJ$ is equal to $\dfrac{1}{6}$

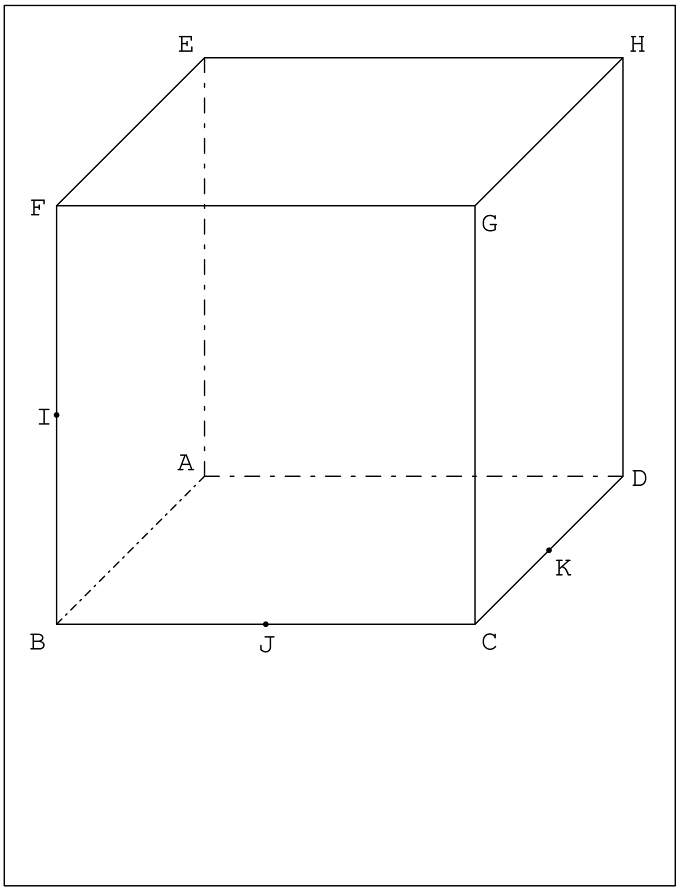
E.4046 Consider a cube $ABCDEFGH$ of edge length 1. We denote by I the middle of $[EF]$ and by J the symmetrical of E with respect to F.



Throughout the exercise, space is referred to the orthonormal reference frame $\left(A\,;\,\overrightarrow{AB}\,;\,\overrightarrow{AD}\,;\,\overrightarrow{AE}\right))$.

1. a. Determine the coordinates of points I and J.
   b. Verify that the vector $\overrightarrow{DJ}$ is a normal vector to the plane (BGI).
   c. Deduce a standard form of the plane (BGI).
   d. Calculate the distance from point F to plane (BGI). `(excluding 2012 program)`.
2. Note (Δ) the straight line passing through F and orthogonal to the plane (BGI).
   a. Give a parametric representation of the line (Δ).
   b. Show that the straight line (Δ) passes through the center K of the face $ADHE$.
   c. Show that the straight line (Δ) and the plane (BGI) intersect at a point, denoted L, with coordinates $\left(\dfrac{2}{3}\,;\dfrac{1}{6}\,;\dfrac{5}{6}\right)$.
   d. *In this question, any trace of research, however incomplete, will be taken into account in the assessment.*
   Is the point L the orthocenter of the triangle BGI?

E.6882 $ABCDEFGH$ denotes a cube with side 1.
The point I is the middle of the segment $[BF]$.
Point J is the middle of segment $[BC]$.
Point K is the midpoint of segment $[CD]$.



Part A

In this part, no justification is required

It is assumed that the straight lines (IJ) and (CG) intersect at a point L.
Construct, on the figure provided above and leaving the construction lines visible:

- the point L;
- the intersection $\mathcal{D}$ of the planes (IJK) and (CDH);
- the section of the cube through the (IJK) plane.

Part B

Space is referred to the reference frame $\left(A\,;\,\overrightarrow{AB}\,;\,\overrightarrow{AD}\,;\,\overrightarrow{AE}\right)$.

1. Give the coordinates A, G, I, J and K in this frame of reference.
2. a. Show that the vector $\overrightarrow{AG}$ is normal to the plane (IJK).
   b. Deduce a standard form of the plane (IJK).
3. We denote by M a point of the segment $[AG]$ and t the real of the interval $[0\,;1]$ tel que $\overrightarrow{AM}=t\cdot\overrightarrow{AG}$.
   a. Demonstrate that : $MI^2=3\cdot t^2-3\cdot t+\dfrac{5}{4}$
   b. Demonstrate that the distance MI is minimal for the point $N\left(\dfrac{1}{2}\,;\dfrac{1}{2}\,;\dfrac{1}{2}\right)$.
4. Demonstrate that for this point $N\left(\dfrac{1}{2}\,;\dfrac{1}{2}\,;\dfrac{1}{2}\right)$:
   a. N belongs to the (IJK) plane.
   b. The straight line (IN) is perpendicular to the straight lines (AG) and (BF).

