



1. Reminders on simplification

E.1081 Reduce, if possible, the following literal expressions :

- (a) $12 + 5x$ (b) $4 \times 5x$
 (c) $3x - (-3) \times x$ (d) $-7x + 9x^2$
 (e) $2x^2 + 7x^2$ (f) $9x^2 + 5x - 4 - 2x + 5x^2$

E.4521 Expand, then reduce the following

expressions :

- (a) $3 \times (2x + 1) - 3x \times 5$ (b) $2(x - 1) + x(2 - x)$
 (c) $-5 \times 3x + 3 \times (5 - x)$ (d) $x(2 - x) + 3(3 + x)$

E.7929 Factor the following expressions :

- (a) $3x + 3$ (b) $6x + 9$ (c) $3x^2 + 2x$ (d) $x^2 + x$

2. The opposite of a number

- E.1715 What are the numbers -4 and 4 called, between them?
 (b) What are the numbers 3 and $\frac{1}{3}$ called, between them?

Definition :

- Two numbers a and b are said to **opposés** if their sums are 0
- Two numbers a and b are said **inverses** if their product is 1.

Note :

- two numbers are opposite 'they have the same distance to zero but are of opposite signs.
- two rational numbers are inverse if their numerator and denominator are reversed between each other.

- (2) Complete, line by line, the table below :

a	b	$a + b$	$a - b$	$-a + b$	$-a - b$
2	3				
-4	5				
9	-2				

- (3) Using the previous table as a guide, complete the following sentences :
- For any number a and b , the opposite of $(a+b)$ is :
 $-(a+b) = \dots\dots\dots$
 - For any number a and b , the opposite of $(a-b)$ is :
 $-(a-b) = \dots\dots\dots$

E.1074

Proposition : the opposite of a sum is equal to the sum of the opposites.

Examples : $-(5+4) = -5-4$ $-(7-3) = -7+3$

For each question, name the number "intruder" :

	(a)	(b)	(c)
(1)	$-(5+1)$	$-5+1$	$-5-1$
(2)	$7-3$	$-(7-3)$	$-7+3$
(3)	$-(-2.5+3)$	$2.5-3$	$2.5+3$
(4)	$-(x-2)$	$-x-2$	$-x+2$

E.1860 For each question, give the intruder value :

	(a)	(b)	(c)	(d)
(1)	$-(2 \times 3)$	$(-2) \times 3$	$2 \times (-3)$	$(-2) \times (-3)$
(2)	$-[(-4) \times 5]$	$4 \times (-5)$	$-4 \times (-5)$	4×5
(3)	$-(2 \times 3 \times 4)$	$(-2) \times 3 \times 4$	$2 \times 3 \times (-4)$	$-2 \times (-3) \times 4$

Note : to take the opposite of a product, we change only one of the factors to its opposite

E.1918 Complete the following dotted lines to verify equality :

- (a) $-(3+4) = \dots 3 \dots 4$ (b) $-(-2+1) = \dots 2 \dots 1$
 (c) $-(-4-7) = \dots 4 \dots 7$ (d) $-(7-9) = \dots 7 \dots 9$

E.7923 Complete the following blanks to verify equality :

- (a) $-(-4+12) = \dots 4 \dots 12$ (b) $-(3+7) = \dots 3 \dots 7$

E.1736 For each question, look for the odd one out:

	a	b	c
1	$-(2 \times 5 + 3)$	$-2 \times 5 + 3$	$-2 \times 5 - 3$
2	$-(3 - 5 \times 2)$	$-3 + 5 \times 2$	$-3 - 5 \times 2$
3	$-(2 \times 3 + 4 \times 5)$	$(-2) \times 3 + 4 \times 5$	$-2 \times 3 - 4 \times 5$

E.1076 Which of the 4 expressions below is

the odd one out?

- a $-[3 \times (2x - 3)]$ b $(-3) \times (2x - 3)$
 c $3 \times (-2x + 3)$ d $(-3) \times (-2x + 3)$
E.7920 Which of the 4 expressions below is the odd one out?
 a $-[2 \times (4x + 1)]$ b $(-2) \times (4x + 1)$
 c $2 \times (-4x - 1)$ d $(-2) \times (-4x - 1)$

3. Removal of parentheses in a sum or difference

E.1737 Give the reduced form of each of the following expressions:

- a $-(2x + 1)$ b $3 - (5 - x)$
 c $2 - (2x - 1)$ d $3x - (-2x - 1)$

E.1919 Give the contracted form of each of the following expressions:

- a $3 - (2 + x) + 5x$ b $x^2 + 2 - (2 \times x + 1)$

E.1923 Give the reduced form of each of the following expressions:

- a $3 - (2x + 1 - x^2)$ b $(x + 1) - (2 - x)$

E.4475 Give the reduced form of each of the following expressions:

- a $(3x + 4) - (x^2 - 4x + 2)$ b $-(x + 3) + x^2 - x + 2$
 c $-(x^2 - 2) + (3x^2 + 4x)$

E.8974 Give the reduced form of each of the following expressions:

- a $(x^2 + 3x + 4) - (5x^2 + 6x + 7)$
 b $-(2x - 5x + 1 - 4 + 7x)$
 c $(3x + 2) - 5x + 6 - (-6x + 2)$

E.8972 Give the reduced form of each of the following expressions:

- a $3x - (x^2 + 4) - 5x + 5$ b $-(x - 2) + (3 - x) + 5x$

E.8970 Give the reduced form of each of the following expressions:

- a $2x^2 + 5 - (2x - 5)$ b $5 - (2x - 4) - 2x^2 + x$

E.8973 Give the reduced form of each of the following expressions:

- a $-(-3x - 1) - (5x - 2)$
 b $-(3x^2 + 4x - 8) - (2x - 4)$

E.7935 Reduce the following expressions:

- a $-(3x^2 + 5x - 9) - (-5x + 1)$
 b $-(x - 2) + 2x + 2 - (3 - x^2)$

E.1946 Give the reduced form of each of the following expressions:

- a $-(2x + x^2 - 4) - (-2x^2 + 4x - 3)$
 b $-(2x^2 - 2) + (x - 2) - (2 - x)$
 c $3x - (2x^2 + 1 - 8x) - (3x - 2x^2 + 7)$

E.8971 Give the reduced form of each of the following expressions:

- a $+(x + 1) - (3x^2 + 2) + [3 - (2 + x)]$
 b $2 - [3 - (x - 2)]$
 c $-[2 - (1 - x) + 1] - (3 - 2x)$

E.4598 Copy and complete the table below, linking the expressions in the left-hand table with their reduced expanded forms obtained in the right-hand table.

$(5x + 5) + 2(4x + 1)$	$\bullet 3x + 3$
$(4x - 6) - (3 - x)$	$\bullet 3x - 3$
$x(2x - 3) + 2(1 - x^2) - 5$	$\bullet 3x - 3$
$(5 - 2x) + x(1 - x) + 8 + x^2$	$\bullet 3x + 3$

4. Parenthesis and simple distributivity

E.1075 Expand and reduce the following expressions:

- a $-3 \times (2 - x)$ b $-2(x + 2)$
 c $-2x(x - 2)$ d $-2 \times (3x - 1)$

E.9006    Expand, then reduce the following expressions:

(a) $-2(x+5)$ (b) $-x(x+2)$

E.1929    Expand and reduce the following expressions:

(a) $3 - 2 \times (2x - 1) + x$ (b) $7 \times (x + 2) - 3 \times (1 + x)$
 (c) $2 \times (x - 1) - 4(x - 3)$ (d) $-2 \times (x - 1) + 4(2x - 3)$

E.8968    Expand, then reduce the following expressions:

(a) $3 - 4 \times (3 - x)$ (b) $3 \times (4x - 2) - 2 \times (3 - x)$

E.9007    Expand, then reduce the following expressions:

(a) $5 - 2(2x + 4)$ (b) $2(x + 1) - 3(3 - x)$

E.8969    Expand, then reduce the following expressions:

(a) $3(x - 2) - 3(2x - 1)$ (b) $(-2 + 3x)(-2) - (6x + 2)$
 (c) $3(5 - 2x) - 2(3 + 2x)$ (d) $(x - 2) \times 2 - (2x + 5)$

E.4549    Expand, then reduce the following expressions:

(a) $3x(x+2) - (-3x^2 - 4x)$ (b) $2(3-x+2) - 2x(3-2x)$

E.1082    Expand and reduce the following expressions:

(a) $3x(2x - 4) - 5(4 - x)$ (b) $-(x + 2) + 3(2x^2 + 1)$

E.8976    Expand, then reduce the following expressions:

(a) $4x(3x - 4) + 2(x^2 + 2)$ (b) $(x^2 + 4x + 3) - (3 - x^2)$

E.4474    Expand, then reduce the following expressions:

(a) $3(x^2 + 4x + 1) + 2(x^2 - 1)$ (b) $5x - 2 - (x^2 + 2)$

(c) $2(-x^2 + 5x + 4 + x) - (3x - 7)$ (d) $-2x(x + 1) - 2(3x - 5)$

E.8967    Expand, then reduce the following expressions:

(a) $x \times (2 - x) - 3 \times (x^2 - 1)$ (b) $2 - (x + 1) \times x$

(c) $-(2 + x) \times 3 + x \times (-x + 1)$ (d) $-x \times (2x - 4) - (3 - 2x^2)$

E.8975    Expand, then reduce the following expressions:

(a) $-2[3 - 7 \times (2 - x)]$ (b) $-2[2 - (1 - x)]$

(c) $1 - 2x[1 - (x + 1)]$

5. Equality of literal expressions

E.1927   

Method definition: To show that two expressions are **different**, we show that for a value of x , their evaluation gives distinct values.

This number is then called a **counterexample to the equality**.

Establish that the following equalities are false:

(a) $(x + 1)(2x - 1) = x^2 + x$
 (b) $3 - (x^2 + x) = (3x + 1)(3 - x)$
 (c) $x^2 + x + 4 = (5x + 1)(4 - 5x)$

E.7927   

Definition: Two expressions are **equal** if they have the same value, regardless of the value of x (This is referred to as **identity**).

Method: To establish an identity, we expand and simplify each expression to show that they have the **same expanded simplified expression**.

Establish that the following identities are true:

(a) $(2x - 1)(1 - x) = -2x^2 + 3x - 1$
 (b) $(3x + 1)(2x - 2) = 6x^2 - 2(2x + 1)$

E.4600    Consider the following two expressions:

$A = 3x^2 + 5x + 1$; $B = (2x + 1)^2$

Justify that the expressions A and B are not equal.

E.4490    Consider the two expressions below:

$A = 2x^2 + x - 7$; $B = 3(x - 2) + 3$

1 (a) Evaluate the expressions A and B for $x = 2$.

(b) Evaluate expressions A and B for $x = -1$.

2 Are the two expressions A and B equal for all values of x ? Justify your assertion.

E.7928    Consider the following two expressions:

$C = (3x - 2)(1 + 2x)$; $D = x \times (6x - 1) - 2$

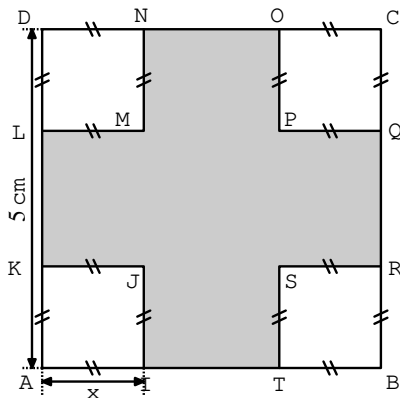
Using expansions and simplifications, show that the expressions C and D are equal.

E.4601   

Consider the figure opposite:

où les quadrilatères $ABCD$, $AIIK$, $BRST$, $COPQ$ and $DLMN$ are squares. Information on the lengths is given in the figure.

The polygon $IJKLMNOPQRST$ is designated $\mathcal{P}$.



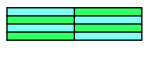
Part A

- ① We're interested in the special case where $x=2$.

6. Usmath

E.9647    

With American notations: to perform double distributivity, we can use a table to represent each of the products of the F.O.I.L. For example, to expand the expression $(2x+3)(x+6)$, we use the table:

	$2x$	3
x		
4		

We obtain the following development:

$$\begin{aligned} 2x+3 \quad x+4 &= \begin{matrix} \text{F.} \\ \text{I.} \end{matrix} \begin{matrix} \text{O.} \\ \text{L.} \end{matrix} \begin{matrix} \text{F.} \\ \text{I.} \end{matrix} \begin{matrix} \text{O.} \\ \text{L.} \end{matrix} \\ &= 2x^2 + 8x + 3x + 12 \\ &= 2x^2 + 11x + 12 \end{aligned}$$

- ① (a) Complete the table 1
(b) Deduce the reduced expansion form of the expression:
 $A = (2x+1)(x+4)$
- ② (a) Complete the table 2
(b) Deduce the reduced expansion form of the expression:
 $B = (x-3)(3x-1)$

	$2x$	1
x		
4		

Tableau 1

	$2x$	1
x	$2x^2$	x
4	$8x$	4

Tableau 1

7. Unclassified exercises

- (a) Determine the areas of squares $ABCD$ and $DLMN$.
(b) Deduct the area of the polygon $\mathcal{P}$ for the value $x=2$.

- ② Justify that the area $\mathcal{A}$ of the polygon $\mathcal{P}$ is expressed as a function of x by: $\mathcal{A}=25-4x^2$

Part B

- ③ We're interested in the special case where $x=1$.

- (a) Determine the areas of the rectangles $KLQR$ and $MPSJ$.

- (b) Deduct the area of the polygon $\mathcal{P}$ for the value $x=1$.

- ④ Justify that the area of the polygon $\mathcal{P}$ is expressed as a function of x by: $\mathcal{A}=10(5-2x)-(5-2x)(5-2x)$

Part C

- ⑤ Justify that the expressions obtained in questions ② and ④ are equal.

E.9646    

With American notation: double distributivity is associated with the acronym F.O.I.L. whose four letters represent the four products:

$$\begin{aligned} a+b \quad c+d &= \begin{matrix} \text{F.} \\ \text{O.} \end{matrix} \begin{matrix} \text{I.} \\ \text{L.} \end{matrix} \\ &= \begin{matrix} a \quad c \\ + \quad a \quad d \\ + \quad b \quad c \\ + \quad b \quad d \end{matrix} \end{aligned}$$

product of the First terms
product of the Outer terms
product of the Inner terms
product of the Last terms

Complete the table below to obtain the four terms obtained by double distributivity:

	F. terms	O. terms	I. terms	L. terms
$(3x+2)(x+4)$				
$(x-2)(2x+1)$				
$(3-x)(5x-2)$				

E.9652    

With American notation: Algebraic expressions are sometimes multiplied in columns, as shown below:

$$\begin{aligned} &\begin{matrix} 2x+3 \\ \times) \quad x+5 \\ \hline 10x+15 \\ +) \quad 2x^2+3x \\ \hline 2x^2+13x+15 \end{matrix} \quad \begin{matrix} 2x-3 \\ \times) \quad 2x^2-4x+3 \\ \hline 6x-9 \\ -8x^2+12x \\ \hline 4x^3-6x^2 \\ +) \quad 4x^3-14x^2+18x-9 \end{matrix} \end{aligned}$$

Write your operations in a row and perform the following multiplications:

- (a) $(2x+1)(3x-2)$ (b) $(3x^2-5x-1)(3x-2)$
(c) $(5-2x)(x^2+1)$ (d) $(x+1)(-3x^2+2x-1)$

E.8924 
expression :



Expand and reduce the following

$$(-3x) \times (2 - x) + 3 \times (x^2 + 3)$$