ChingQuizz : 4 exercises available for Quizz assessment :

1. *Reminders about powers*

E.11261 Perform the following calculations :

(a) $5-2\times6^2$ (b) $2\times(30-5^2)$ (c) $4+(18-12)^2$

E.11262

Proposal :

- To multiply two powers of the same number, add the exponents.

  Example : $8^2\times8^3=8\times8\times8\times8\times8=8^5$

- To divide two powers of the same number, add the exponents.

  Example : $\dfrac{7^5}{7^3}=\dfrac{7\times7\times7\times7\times7}{7\times7\times7}=\dfrac{7\times7\times\cancel{7}\times\cancel{7}\times\cancel{7}}{\cancel{7}\times\cancel{7}\times\cancel{7}}=7^2$

Simplify the following expressions :

(a) $3^5\times3^8$ (b) $\dfrac{7^{10}}{7^6}$ (c) $6^8\times6^4$ (f) $\dfrac{6^7}{6^4}$

E.11263 Simplify the writing of the following expressions :

(a) $3^2\times3^4$ (b) $5^8\times5^7$ (c) 3×3^4

(d) $\dfrac{3^5}{3^2}$ (e) $\dfrac{8^3}{8^2}$ (f) $\dfrac{7^{12}}{7^5}$

E.11270

Definition : For any non-zero number a $(a\neq0)$ and for any positive integer n, we note : $\dfrac{1}{a^n}=a^{-n}$

$a^0=1$; $a^1=a$

Examples :

$\dfrac{5^3}{5^7}=\dfrac{1}{5^4}=5^{-4}$; $7^{-2}\times7^6=\dfrac{1}{7^2}\times7^6=\dfrac{7^6}{7^2}=7^{6-2}=7^4$

Complete the following calculations :

(a) $\dfrac{5^4}{5^7}=5^{\cdots}=\dfrac{1}{5^{\cdots}}$ (b) $7^4\times7^{-10}=7^{\cdots}=\dfrac{1}{7^{\cdots}}$

(c) $\dfrac{3^{13}}{3^{20}}=3^{\cdots}=\dfrac{1}{3^{\cdots}}$ (d) $8^{-5}\times8^2=8^{\cdots}=\dfrac{1}{8^{\cdots}}$

E.11328 Simplify the following expressions :

(a) $3^5\times3^3\times3^8$ (b) $\dfrac{7^{12}}{7^5}$

(c) $\dfrac{5^7\times5^4}{5^9}$ (d) $\dfrac{9^2\times9^3}{9^5\times9^{10}}$

E.11264 Simplify the writing of expressions :

(a) $\dfrac{10\times10^{-4}}{10^{-8}}$ (b) $\dfrac{10^5\times10^{-4}}{10^{-3}}$ (c) $\dfrac{10^{-12}\times10^8}{10^4}$

E.11265 Simplify the writing of expressions :

(a) $\dfrac{10^3}{(10^{-2})^4}$ (b) $\dfrac{10^{16}}{(10^2)^8}$ (c) $\left(\dfrac{10^5}{10^7}\right)^2$ (d) $(10^3)^{-2}$

E.11266

Proposition : for a a positive integer, m and n two strictly positive integers :

$a^n\times a^m=a^{n+m}$; $\dfrac{a^n}{a^m}=a^{n-m}$; $(a^n)^m=a^{n\times m}$

Simplify the following powers :

(a) $5^2\times5^5$ (b) $7^4\times7^{-7}$ (c) 5×5^{-4}

(d) $3^5\times(3^2)^4$ (e) $8^5\times8^{-3}\times8^{-2}$ (f) $5^{20}(\times5^{-9})^3$

2. *Euclidean division*

E.11333 Perform the following Euclidean divisions :

(a) 132 by 8 (b) 213 by 6

Note : the results will be written in the form :
Dividende = Quotient×Diviseur + Reste

E.9853

1. Complete the blanks below using integers :

   (a) $87=\ldots\times4+7$ (b) $87=\ldots\times4+3$

   (c) $87=\ldots\times4-1$ (d) $87=23\times4+\ldots$

2. Which of the following equalities represents the Euclidean division of 87 by 4?

E.2155 Which of the following equalities represents the Euclidean division of 375 by 14?

(a) $375 = 25\times14 + 25$ (b) $375 = 26\times14 + 11$

(c) $375 = 27\times14 - 3$

E.9852

(1) Complete the blanks below using integers:

(a) $148 = \ldots\times13 + 5$ (b) $148 = \ldots\times13 + 18$

(c) $148 = \ldots\times13 - 8$ (d) $148 = 13\times13 + \ldots$

(2) Which of the following equalities represents the Euclidean division of 148 by 13?

E.9851 For each question, determine the equality expressing the Euclidean division of a by b:

(a) $a = 370$; $b = 250$ (b) $a = 315$; $b = 16$

(c) $a = 1\,254$; $b = 26$ (d) $a = 24\,576$; $b = 134$

E.9870 For each question, determine the equality expressing the Euclidean division of a by b:

(a) $a = 29$; $b = 29$ (b) $a = 65$; $b = 120$

3. *Divisors of a number*

E.9854

(1) (a) Give the six factors of the number 12.

(b) Give all the factors of the number 27.

(2) What are the common factors of the integers 12 and 27.

E.2133 *In this exercise, any evidence of research, even if incomplete, or of initative, even if unsuccessful, will be considered in the evaluation.*

"*The hidden number:*

- *I am a whole number between 100 and 400.*
- *I am even.*
- *I am divisible by 11.*
- *I also have 3 and 5 as factors.*

Qui suis-je?".

Explain a process for finding the hidden number, and give its value.

E.9149

(1) Give the eight factors of the integer 30 and the eight factors of the integer 24.

(2) What are the common factors of the integers 30 and 24?

E.4998 Complete the table below:

Entier x	2	3	4	5	6	7	8	9	10	11	12
Nombre de diviseurs de x	2	2	3								

E.10682 Which of the triplets of numbers below contains three divisors of the integer 144:

(a) $(2\,;9\,;4)$ (b) $(2\,;3\,;7)$ (c) $(4\,;5\,;9)$

4. *Integer parity*

E.10902 Complete the following two double-entry tables:

+	Even	Odd
Even		
Odd		

×	Even	Odd
Even		
Odd		

E.10903 Complete the following sentences without justification using the words **even**, **odd**, **any**.

(a) The sum of two even integers is an integer ……

(b) The sum of two odd integers is an integer ……

(c) The product of two odd integers is an integer ……

(d) The product of an even integer by an odd integer is an integer ……

E.10904 Without justification, say whether the following assertions are true, false or undecidable:

(1) The sum of two odd integers is an even integer.

(2) The product of an even integer and an odd integer is even.

(3) The product of two consecutive integers is an even integer.

(4) The sum of five consecutive integers is a multiple of 5.

E.10872

Consider the following calculation program :

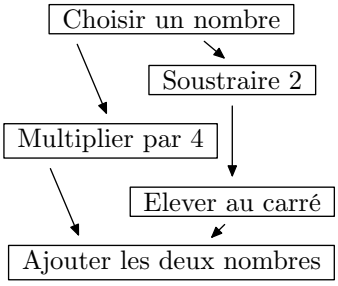
- Choisir un nombre
- Calculer son carré
- Ajouter 6 au résultat

Are the following statements true or false? Justify your answers and write down the steps for any calculations :

1. "*If we choose the number* $\frac{2}{3}$, *the result of the program is* $\frac{58}{9}$. "
2. "*If an integer is chosen, the result of the program is an odd integer.*"
3. "*The result of the program is always a positive number.*"
4. "*If the chosen number is an even integer, then the result is an even integer.*"

E.10901

Consider the calculation program below :



1. "*If we choose the number* $\frac{2}{3}$, *the result of the program is* $\frac{40}{9}$."
2. "*If an integer is chosen, the result of the program is an odd integer.* "
3. "*The result of the program is always a positive number.*"
4. "*If the chosen integer is an even integer then the result is an integer pair*"

5. *Prime numbers*

E.8008

Definition : An integer a is said to be **prime** if it has exactly two divisors *(1 and itself)*

In the list of numbers below, cross out the numbers that are not prime integers :

33	47	51	28	39	49	85

E.10445 Justify that each of the integers below is not a prime integer :

573 ; 1784 ; 1065

E.8003 Indicate and justify whether the following statement is true or false.

Assertion : "*The number* 231 *is a number premier*"

E.8004 *In this exercise, one question is asked and only one of the four answers provided is correct. State the correct answer and justify your choice.*

The numbers 23 and 37 :

a) are first
b) are divisible by 3.
c) have no common factors
d) are even.

E.8009 *Tell whether the statement below is true or false, justifying the answer.*

Assertion : for any positive integer n, the number $2n+1$ is a prime number.

E.10714 Which of the following integers is a prime integer :

16 993 ; 16 984 ; 17 007 ; 16 983 ; 16 985

E.10722 Which of the following integers is a prime number :

17 401 ; 17 396 ; 17 391 ; 17 409 ; 17 395

E.8002 *Say whether the statement, with justification, is true or false.*

Assertion : For all integers n between 2 and 9, the integer 2^n-1 is a prime.

E.9855

1. Give three pairs of prime integers whose sum is a prime integer.
2. Consider a pair $(a\,;b)$ of prime integers such that $a+b$ is a prime integer.
   Study the parity of the integers a and b.

6. *Prime factorization (with 2, 3, 5, 7)*

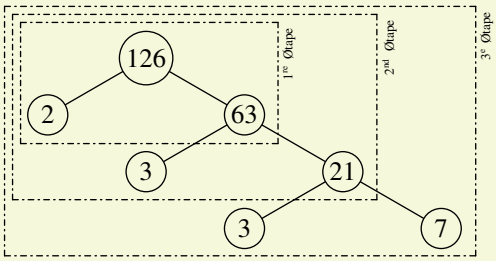
E.9265 The number 588 can be decomposed as :

$588=2^2\times3\times7^2$

What are its prime factors? That is, numbers that are both primes and factors of 588.

E.9856

Example : to determine the decomposition of the integer 126 into a product of prime factors, we use the following algorithm which is schematized in the diagram below :



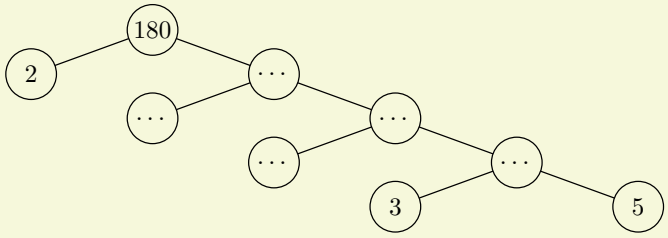
- We look for a prime factor of 126 : for example 2.
  We obtain the equality : $126 = 2\times 63$
- We look for a prime factor of 63 : for example 3.
  We obtain the equality : $126 = 2\times 3\times 21$
- We look for a prime factor of 21 : for example 3.
  We obtain the equality : $126 = 2\times 3\times 3\times 7$
- We stop since 7 is also a prime number.

Determine the product of prime factors decomposition of the integers below :

(a) 45 (b) 140 (c) 196

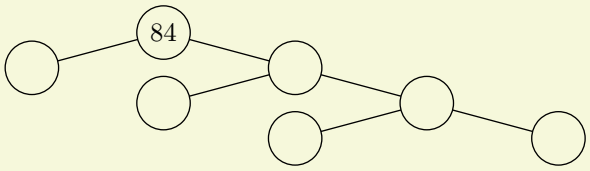
E.5678 Complete the diagram below to obtain the prime factorization of the integer 180.

The diagram can be used as a guide :



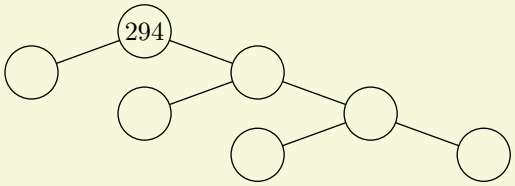
E.11297 Give the prime factorization of the integer 84.

The diagram can be used to help :



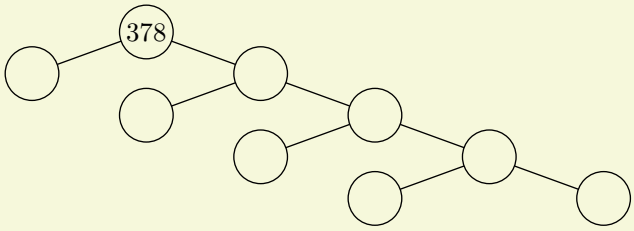
E.10728 Give the prime factorization of the integer 294

The diagram can be used to help :



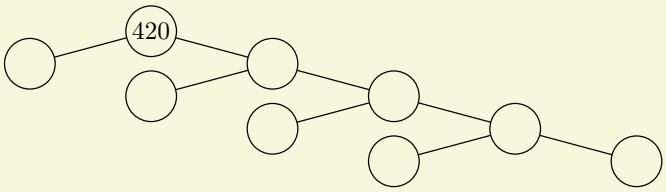
E.10727 Give the product of prime factors decomposition of the integer 378

The diagram can be used as a guide :



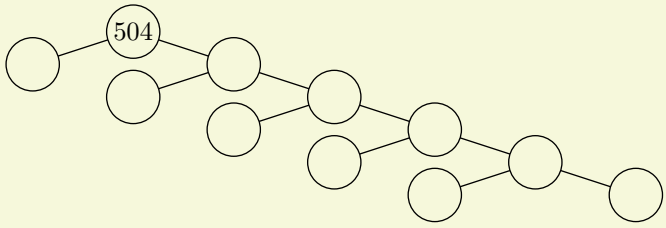
E.11295 Give the prime factorization of the integer 420.

The diagram can be used to help :



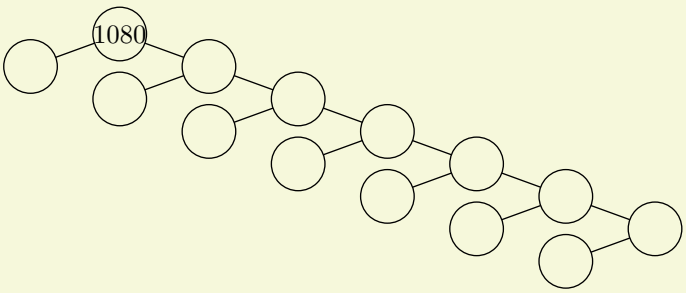
E.11412 Give the prime factorization of the integer 504 :

The diagram can be used to help :



E.11296 Give the prime factorization of the integer 1080.

The diagram can be used to help :



E.9858

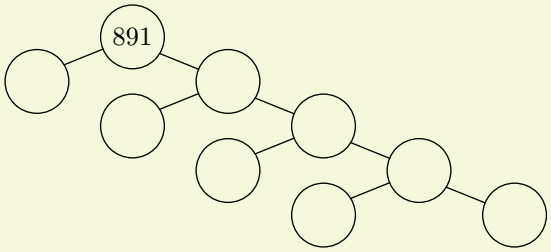
(1) Determine the prime factor decomposition of 27 000 000.

(2) What are its prime factors?

7. *Prime factor decomposition*

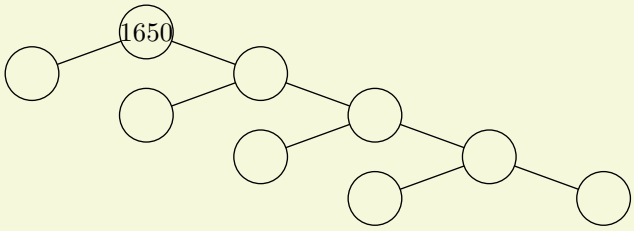
E.10723 Give the prime factorization of the integer 891 :

The diagram can be used to help :



E.10726 Give the prime factor product decomposition of the integer 1650 :

The diagram can be used as a guide :



8. *Operations on prime factor decompositions*

E.10692 Let a and b be two integers, in each of the cases below, give the prime factor product decomposition of the integer $a \times b$:

(a) $a = 2^3 \times 3 \times 5^2$ et $b = 3^5 \times 5$

(b) $a = 3^4 \times 5 \times 7^2$ et $b = 2 \times 3^3$

E.11260

1. Give the prime factorization of the integers :
   175 ; 108
2. Noting that $175 \times 108 = 18\,900$, give the prime factorization of 18 900.

E.8010 Determine the product of prime factors decomposition of each of the numbers below :

(a) 14×12 (b) 35×24 (c) 16×54

E.10701

1. Give the prime product decomposition of the integers 162 and 270.
2. Deduce the prime product decomposition of the integer 162×270.

9. *Quotient of factorizations into prime factors*

E.10672 For each of the fractions below, give their irreducible expression :

(a) $\dfrac{2^4 \times 3^8 \times 7^3}{2^7 \times 3^6 \times 7^2}$ (b) $\dfrac{2^{12} \times 3^4 \times 5^5}{2^8 \times 3^4 \times 5^7}$ (c) $\dfrac{2^{34} \times 5^{17}}{2^{30} \times 3^3 \times 5^{15}}$

E.10699 Let a and b be two integers. For each question, give the prime factorization of the integer $\dfrac{a}{b}$:

(a) $a = 2^3 \times 3 \times 5^3$ et $b = 2^2 \times 3 \times 5$

(b) $a = 2^4 \times 3^4 \times 7^2$ et $b = 2 \times 3^3$

E.10724 Consider the two integers :
$a = 2^6 \times 3 \times 5^2 \times 7^2$; $b = 2^4 \times 5^2 \times 7$

For each question, give the prime factorization of the calculation :

(a) $a \times b$ (b) $\dfrac{a}{b}$

E.10702

1. Give the prime product decompositions of the integers 2160 and 36.
2. Deduce the prime product decomposition of the integer $\dfrac{2160}{36}$.

E.11330

1. Give the prime factorization : (a) of the integer 364
2. Deduce the prime factorization of 364×378.
3. Give the reduced form of the fraction $\dfrac{364}{378}$.

E.11413

1. Give the prime factorization of the integers 180 and 378.
2. Deduce the prime factorization of the integer 180×378.
3. Deduce the reduced expression of the fraction $\dfrac{180}{378}$.

E.10673

1. Give the prime factor product decomposition of the integers 12, 44 and 72.
2. Deduce the irreducible expression of the fraction $\dfrac{12 \times 44}{72}$.

10. *Factorization of decompositions into prime factors*

E.10704 Establish that the integers below are multiples of 5:

a) $2^{155}\times3^{78}+2^{154}\times3^{79}$

b) $2^{64}\times3^{23}\times7^{49}+2^{62}\times3^{24}\times7^{50}$

E.10725 Establish that the integer A is a multiple of 7:

$2^{53}\times3^{40}+2^{55}\times3^{39}$

E.10705 Consider the integer a defined by:

$a=4^{17}\times3^{27}\times5^{17}+2^{31}\times9^{13}\times5^{19}$

Show that the integer a is a multiple of 7.

11. *Deepening: a little arithmetic*

E.9240

1) a) Determine the prime factor product decomposition of 2 744.

b) Derive the prime factor product decomposition of $2\,744^2$.

c) Using this decomposition, find x such that: $x^3=2\,744^2$

2) Let a and b be two integers greater than 2 such that $a^3=b^2$.

a) Calculate b when $a=100$.

b) Determine two integers a and b greater than 2 and less than 10 that verify the equality $a^3=b^2$.

E.9860 The sum of two multiples of 5 is always a multiple of 5.