ChingQuizz : 2 exercises available for Quizz assessment :

1. *Reduced normal distribution*

E.5468 Consider the function f defined on $\mathbb{R}$ by the relation :

$$f(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{t^2}{2}}$$

1. Using the calculator, determine the values rounded to the nearest thousandth of the following integrals :

   a. $\displaystyle\int_{-1}^{1} f(t)\,\mathrm{d}t$ b. $\displaystyle\int_{-2}^{2} f(t)\,\mathrm{d}t$ c. $\displaystyle\int_{-3}^{3} f(t)\,\mathrm{d}t$

   d. $\displaystyle\int_{-4}^{4} f(t)\,\mathrm{d}t$ e. $\displaystyle\int_{-5}^{5} f(t)\,\mathrm{d}t$ f. $\displaystyle\int_{-9}^{9} f(t)\,\mathrm{d}t$

2. What conjecture can be made about the value of the following limit :

$$\lim_{x\mapsto+\infty}\int_{-x}^{x} f(t)\,\mathrm{d}t$$

E.5467 Consider a random variable with a probability distribution $\mathcal{N}(0\,;1)$. Using a calculator, give the values of the following probabilities, rounded to the thousandth :

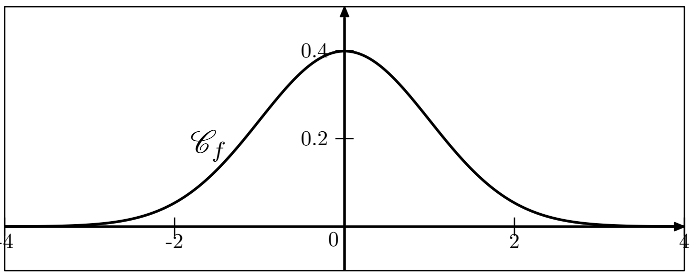
a. $\mathcal{P}(\mathcal{X}\leqslant-1)$ b. $\mathcal{P}(\mathcal{X}\leqslant2)$ c. $\mathcal{P}(\mathcal{X}\geqslant1)$

E.2654 Consider a random variable with a probability distribution $\mathcal{N}(0\,;1)$. Using a calculator, give the following probability values, rounded to the thousandth :

a. $\mathcal{P}(-2\leqslant\mathcal{X}\leqslant3)$ b. $\mathcal{P}(0\leqslant\mathcal{X}\leqslant2)$ c. $\mathcal{P}(-4\leqslant\mathcal{X}\leqslant4)$

E.5470 Consider a random variable following a centered reduced normal distribution $(\mathcal{X}\sim\mathcal{N}(0\,;1))$.

1. Give the value of : $\mathcal{P}(\mathcal{X}\leqslant0)$.
2. Note $\mathscr{C}_f$ the curve representing the density f of the random variable $\mathcal{X}$:



   a. Interpret graphically the number $\mathcal{P}(\mathcal{X}\leqslant2)$.

   b. Using a calculator, give the value of $\mathcal{P}(0\leqslant\mathcal{X}\leqslant2)$ rounded to the nearest thousandth.

   c. Deduct the probability $\mathcal{P}(\mathcal{X}\leqslant2)$, rounded to the nearest thousand.

E.5471 Consider a random variable $\mathcal{X}$ suivant a normal distribution centered reduced $(\mathcal{N}(0\,;1))$. We give the probability rounded to the thousandth :

$$\mathcal{P}\Big(\mathcal{X}\leqslant\frac{1}{2}\Big)\approx0.691$$

Without the calculator, determine the value of the following probabilities :

a. $\mathcal{P}\Big(0\leqslant\mathcal{X}\leqslant\frac{1}{2}\Big)$ b. $\mathcal{P}\Big(-\frac{1}{2}\leqslant\mathcal{X}\leqslant0\Big)$ c. $\mathcal{P}\Big(\mathcal{X}\leqslant-\frac{1}{2}\Big)$

E.6953 Below is a table of values for a variable $\mathcal{X}$ following a reduced centered normal distribution :

t_1 \ t_2	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
-2,0	0,0228	0,0233	0,0239	0,0244	0,025	0,0256	0,0262	0,0268	0,0274	0,0281
-1,9	0,0287	0,0294	0,0301	0,0307	0,0314	0,0322	0,0329	0,0336	0,0344	0,0351
-1,8	0,0359	0,0367	0,0375	0,0384	0,0392	0,0401	0,0409	0,0418	0,0427	0,0436
-1,7	0,0446	0,0455	0,0465	0,0475	0,0485	0,0495	0,0505	0,0516	0,0526	0,0537
-1,6	0,0548	0,0559	0,0571	0,0582	0,0594	0,0606	0,0618	0,063	0,0643	0,0655
-1,5	0,0668	0,0681	0,0694	0,0708	0,0721	0,0735	0,0749	0,0764	0,0778	0,0793
-1,4	0,0808	0,0823	0,0838	0,0853	0,0869	0,0885	0,0901	0,0918	0,0934	0,0951
-1,3	0,0968	0,0985	0,1003	0,102	0,1038	0,1056	0,1075	0,1093	0,1112	0,1131
-1,2	0,1151	0,117	0,119	0,121	0,123	0,1251	0,1271	0,1292	0,1314	0,1335
-1,1	0,1357	0,1379	0,1401	0,1423	0,1446	0,1469	0,1492	0,1515	0,1539	0,1562
-1,0	0,1587	0,1611	0,1635	0,166	0,1685	0,1711	0,1736	0,1762	0,1788	0,1814
-0,9	0,1841	0,1867	0,1894	0,1922	0,1949	0,1977	0,2005	0,2033	0,2061	0,209
-0,8	0,2119	0,2148	0,2177	0,2206	0,2236	0,2266	0,2296	0,2327	0,2358	0,2389
-0,7	0,242	0,2451	0,2483	0,2514	0,2546	0,2578	0,2611	0,2643	0,2676	0,2709
-0,6	0,2743	0,2776	0,281	0,2843	0,2877	0,2912	0,2946	0,2981	0,3015	0,305
-0,5	0,3085	0,3121	0,3156	0,3192	0,3228	0,3264	0,33	0,3336	0,3372	0,3409
-0,4	0,3446	0,3483	0,352	0,3557	0,3594	0,3632	0,3669	0,3707	0,3745	0,3783
-0,3	0,3821	0,3859	0,3897	0,3936	0,3974	0,4013	0,4052	0,409	0,4129	0,4168
-0,2	0,4207	0,4247	0,4286	0,4325	0,4364	0,4404	0,4443	0,4483	0,4522	0,4562
-0,1	0,4602	0,4641	0,4681	0,4721	0,4761	0,4801	0,484	0,488	0,492	0,496
-0,0	0,5	0,504	0,508	0,512	0,516	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,591	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,648	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,67	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,695	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,719	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,758	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,791	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,834	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,877	0,879	0,881	0,883
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,898	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,937	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,975	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817

$\mathcal{P}(\mathcal{X}\leqslant t_1+t_2)$ $\mathcal{X}\sim\mathcal{N}(0\,;1)$

1. Using the table of values, give the following probabilities :

   a. $\mathcal{P}(\mathcal{X}\leqslant0.7)$ b. $\mathcal{P}(\mathcal{X}\leqslant1.47)$ c. $\mathcal{P}(\mathcal{X}\leqslant-0.44)$

2. The probabilities will be rounded to 10^{-3}. Justify your approach and determine the following probabilities :

   a. $\mathcal{P}(\mathcal{X}>1.5)$ b. $\mathcal{P}(\mathcal{X}>-0.37)$

   c. $\mathcal{P}(-1\leqslant\mathcal{X}\leqslant1)$ d. $\mathcal{P}(-0.55\leqslant\mathcal{X}<1.2)$

E.7580 Consider the centered and reduced random variable ($\mathcal{N} \sim \mathcal{N}(0\ 1)$). What is the probability that the random variable $\mathcal{X}$ has a value less than 0.5 knowing that it has a value less than 2?

- 0.706
- 0.707
- 0.708
- 0.709

2. Reduced centered inverse normal distribution

E.5469 Let $\mathcal{X}$ be a random variable following a normal distribution $\mathcal{N}(0\,;1)$. For each question, use a calculator to determine the value of a, rounded to three decimal places, checking that the following equations are true:

(a) $\mathcal{P}(\mathcal{X}\leqslant a)=0.3$ (b) $\mathcal{P}(\mathcal{X}\leqslant a)=0.5$ (c) $\mathcal{P}(\mathcal{X}\leqslant a)=0.68$

E.5472 Consider a random variable following a reduced normal distribution ($\mathcal{X}\sim\mathcal{N}(0\,;1)$).

In each case, use a calculator to determine the value, rounded to the nearest thousandth, of the real number x that satisfies the following equations:

(a) $\mathcal{P}(\mathcal{X}\leqslant x) = 0.65$ (b) $\mathcal{P}(\mathcal{X}\geqslant x) = 0.65$

(c) $\mathcal{P}(-x\leqslant\mathcal{X}\leqslant x) = 0.65$ (d) $\mathcal{P}(-x\leqslant\mathcal{X}\leqslant x) = 0.95$

Note: we will try to reduce this to equations of the form $\mathcal{P}(\mathcal{X}\leqslant x)=\alpha$

3. Normal law and centered-reduced normal law

E.5483 Let $\mathcal{X}$ be a random variable following a normal distribution of parameter μ and σ^2 ($\mathcal{X}\sim\mathcal{N}(\mu\,;\sigma^2)$). We denote $\mathcal{Z}$ a random variable following the centered reduced normal distribution ($\mathcal{Z}\sim\mathcal{N}(0\,;1)$).

By centering and reducing the random variable $\mathcal{X}$, establish the equality:

$\mathcal{P}(\mathcal{X}\leqslant\mu) = 0.5$

E.5478 Let $\mathcal{X}$ be a random variable following a normal distribution with parameters 3 and 4 ($\mathcal{X}\sim\mathcal{N}(3\,;4)$) and $\mathcal{Z}$ a random variable following a reduced centered normal distribution ($\mathcal{Z}\sim\mathcal{N}(0\,;1)$)

1. Express the probabilities below as a function of the random variable $\mathcal{Z}$:

   (a) $\mathcal{P}(\mathcal{X}\geqslant 2)$ (b) $\mathcal{P}(\mathcal{X}<-2)$ (c) $\mathcal{P}(-2\leqslant\mathcal{X}\leqslant 3)$

2. Using the table below and showing your work, determine the probabilities for question (1).

t_1 \ t_2	0,00	0,05	0,10	0,15	0,20
-3,00	0,0013	0,0016	0,0019	0,0022	0,0026
-2,75	0,003	0,0035	0,004	0,0047	0,0054
-2,50	0,0062	0,0071	0,0082	0,0094	0,0107
-2,25	0,0122	0,0139	0,0158	0,0179	0,0202
-2,00	0,0228	0,0256	0,0287	0,0322	0,0359
-1,75	0,0401	0,0446	0,0495	0,0548	0,0606
-1,50	0,0668	0,0735	0,0808	0,0885	0,0968
-1,25	0,1056	0,1151	0,1251	0,1357	0,1469
-1,00	0,1587	0,1711	0,1841	0,1977	0,2119
-0,75	0,2266	0,242	0,2578	0,2743	0,2912
-0,50	0,3085	0,3264	0,3446	0,3632	0,3821
-0,25	0,4013	0,4207	0,4404	0,4602	0,4801
0,00	0,5	0,5199	0,5398	0,5596	0,5793
0,25	0,5987	0,6179	0,6368	0,6554	0,6736
0,50	0,6915	0,7088	0,7257	0,7422	0,758
0,75	0,7734	0,7881	0,8023	0,8159	0,8289
1,00	0,8413	0,8531	0,8643	0,8749	0,8849
1,25	0,8944	0,9032	0,9115	0,9192	0,9265
1,50	0,9332	0,9394	0,9452	0,9505	0,9554
1,75	0,9599	0,9641	0,9678	0,9713	0,9744
2,0	0,9772	0,9798	0,9821	0,9842	0,9861
2,25	0,9878	0,9893	0,9906	0,9918	0,9929
2,50	0,9938	0,9946	0,9953	0,996	0,9965
2,75	0,997	0,9974	0,9978	0,9981	0,9984

$\mathcal{P}(\mathcal{X}\leqslant t_1+t_2)$ $\mathcal{X}\sim\mathcal{N}(0\,;1)$

4. *Normal law*

E.5473 Consider a random variable $\mathcal{X}$ following a normal distribution with parameters 10 and 9 . $(\mathcal{X}\sim\mathcal{N}(10\,;9))$.

1. Give the expectation, variance and standard deviation of the random variable $\mathcal{X}$.
2. Using a calculator, determine the following probabilities rounded to the nearest thousandth:

   a. $\mathcal{P}(\mathcal{X}\leqslant 9)$ b. $\mathcal{P}(0\leqslant\mathcal{X}\leqslant 12)$

E.5474 Consider a random variable $\mathcal{X}$ following a normal distribution with parameters 3 and 4. $(\mathcal{X}\sim\mathcal{N}(3\,;16))$.

1. Give the expectation, variance and standard deviation of the random variable $\mathcal{X}$.
2. We give the probability rounded to the thousandth: $\mathcal{P}(\mathcal{X}\leqslant 4)\approx 0.691$

   Without the aid of a calculator, determine the following probabilities:

   a. $\mathcal{P}(\mathcal{X}\leqslant 3)$ b. $\mathcal{P}(3\leqslant\mathcal{X}\leqslant 4)$

   c. $\mathcal{P}(2\leqslant\mathcal{X}\leqslant 3)$ d. $\mathcal{P}(\mathcal{X}\leqslant 2)$

E.5479 Consider a random variable $\mathcal{X}$ following a normal distribution with parameters -3 et 5 $(\mathcal{X}\sim\mathcal{N}(-3\,;5))$.

1. Give the mean, variance and standard deviation of the random variable $\mathcal{X}$.
2. Without the aid of a calculator, which of the probabilities below have a value strictly greater than 0.5:

   a. $\mathcal{P}(\mathcal{X}\leqslant 0)$ b. $\mathcal{P}(\mathcal{X}\geqslant 2)$ c. $\mathcal{P}(\mathcal{X}>-4)$

E.6960 ⚠ The company "*Bonne Mamie*" uses a machine to fill jam jars on a production line. Note $\mathcal{X}$ the random variable that associates the mass of jam in grams with each jar of jam produced.

Assuming that the machine is correctly set up, we assume that $\mathcal{X}$ follows a normal distribution with mean $\mu=125$ and standard deviation $\sigma=2$.

A jar of jam is considered to conform when the mass of jam it contains is between 120 and 130 grams.

A jar is chosen at random from those with a jam mass of less than 130 grams. What is the probability that this jar does not conform? We'll give the result rounded to 10^{-4}.

E.5484 A delivery company regularly drops off its parcels at the town hall. The driver has two possible routes to the town hall. After surveying several deliveries, the manager makes the following proposals:

- We note $\mathcal{X}$ is the random variable modeling in minutes the duration of the journey **A**. The random variable $\mathcal{X}$ follows a normal distribution with parameters 41 and 25 $(\mathcal{X}\sim\mathcal{N}(41\,;25))$.
- We note $\mathcal{Y}$ is the random variable modeling in minutes the travel time **B**. The random variable $\mathcal{X}$ follows a normal distribution with parameter 43 and 4 $(\mathcal{Y}\sim\mathcal{N}(43\,;4))$.

Packages must be delivered by $12\,h$ in order for the company not to pay a penalty.

1. Leaving the company at $11\,h\ 16\,min$, which route should the deliveryman choose to have the best chance of arriving on time?
2. Leaving the company at $11\,h\ 15\,min$, which route should the delivery driver choose to have the best chance of arriving on time?

5. *Normal law: range and standard deviation*

E.5481 Let $\mathcal{X}$ be a random variable following a normal distribution with parameters -2 and 4. $(\mathcal{X}\sim\mathcal{N}(-2\,;4))$.

Without a calculator, give the following probabilities rounded to two decimal places:

a. $\mathcal{P}(-4\leqslant\mathcal{X}\leqslant 0)$ b. $\mathcal{P}(-6\leqslant\mathcal{X}\leqslant 2)$ c. $\mathcal{P}(-8\leqslant\mathcal{X}\leqslant 4)$

6. *Normal law: determining an interval*

E.5480 Let $\mathcal{X}$ be a random variable following a normal distribution with parameter 2 and 9. $(\mathcal{X}\sim\mathcal{N}(2\,;9))$.

The questions will be answered using the calculator:

1. Determine the value of x, rounded to the nearest hundredth, verifying the equality: $\mathcal{P}(\mathcal{X}\leqslant x)=0.24$
2. Determine the value of x, rounded to the nearest hundredth, verifying the equality: $\mathcal{P}(\mathcal{X}\geqslant x)=0.7$
3. Determine the value of x, rounded to the nearest hundredth, verifying the equality: $\mathcal{P}(2-x\leqslant\mathcal{X}\leqslant 2+x)=0.5$

Express these conditions in the form $\mathcal{P}(\mathcal{X}\leqslant x)$

E.5520 A sleeve is considered to be compliant in terms of diameter when its internal diameter, expressed in millimeters, falls within the range $[8.12\,;8.48]$.

Let $\mathcal{X}$ be the random variable that associates the internal diameter with each sleeve randomly selected from a day's production.

We assume that $\mathcal{X}$ follows a normal distribution with mean 8.33 and standard deviation 0.09.

1. Calculate the probability that a randomly selected sleeve will be compliant in terms of its inner diameter.

We will use the table of values below où The random variable $\mathcal{Z}$ follows a centered and reduced normal distribution. $(\mathcal{Z}\sim\mathcal{N}(0\,;1))$.

k	-3.52	-2.33	-1.96	-0.45	0.32	1.24	1.67	1.96
$\mathcal{P}(\mathcal{Z}\leqslant k)$	0.004	0.009	0.025	0.326	0.626	0.893	0.953	0.975

2. Calculate the positive real number h such that:
$\mathcal{P}(8.33-h\leqslant\mathcal{X}\leqslant 8.33+h) = 0.95$
Interpret the result in a sentence.

7. *Normal law: finding a parameter*

E.5512 The company *Fructidoux* manufactures compotes that it packages in small 50 gram jars. It wants to label them as "*low-calorie compote*".

The law requires that the sugar content, i.e., the proportion of sugar in the compote, be between 0.16 and 0.18. In this case, the small jar of compote is said to be compliant.

We denote $\mathcal{X}$ the random variable that associates the sugar content with a small jar taken at random from production.

We assume that $\mathcal{X}$ follows a normal distribution with mean $m=0.17$ and standard deviation σ.

We also assume that the probability that a small jar taken at random from production is compliant is equal to 0.99.

Let $\mathcal{Z}$ be the random variable defined by: $\mathcal{Z}=\dfrac{\mathcal{X}-m}{\sigma}$.

1. What distribution does the random variable $\mathcal{Z}$ follow?
2. Determine, as a function of σ the interval to which $\mathcal{Z}$ belongs when $\mathcal{X}$ appartient to the interval $[0.16\,;0.18]$.
3. Determine the value of σ rounded to 10^{-3} près.

We can use the table below, in which the random variable $\mathcal{Z}$ follows the normal distribution of expectation 0 and standard deviation 1.

$\mathcal{P}(-\beta\leqslant\mathcal{Z}\leqslant\beta)$	β
0.985	2.4324
0.986	2.4573
0.987	2.4838
0.988	2.5121
0.989	2.5427

$\mathcal{P}(-\beta\leqslant\mathcal{Z}\leqslant\beta)$	β
0.990	2.5758
0.991	2.6121
0.992	2.6521
0.993	2.6968
0.994	2.7478

E.5485 Consider a random variable $\mathcal{X}$ following a normal distribution with parameter 0 and σ^2. $(\mathcal{X}\sim\mathcal{N}(0\,;\sigma^2))$.

We seek to determine the value of σ so that we have the following probability:
$\mathcal{P}(\mathcal{X}\leqslant 4) = 0.7$

We denote $\mathcal{Z}$ a random variable following a normal distribution with parameters 0 and 1. $(\mathcal{Z}\sim\mathcal{N}(0\,;1))$

1. Determine the value of x, rounded to the nearest thousandth, such that: $\mathcal{P}(\mathcal{Z}\leqslant x) = 0.7$
2. Deduce a value approximating the standard deviation σ to the nearest thousandth.

E.6954 A company manufactures spherical wooden logs.

The diameter of a log taken at random from production is modelled using a random variable $\mathcal{Y}$ which follows a normal distribution with expectation $\mu=1$ and standard deviation σ, σ being a strictly positive real.

Knowing that $\mathcal{P}(0.9\leqslant\mathcal{Y}\leqslant 1.1)=0.98$, determine the value of σ, rounded to the nearest thousandth.

E.6963 The duration of parking for a car in a downtown parking lot is modeled by a random variable T that follows a normal distribution with mean μ and standard deviation σ.
We know that the average parking time in this parking lot is 30 minutes and that 75 % of the cars have a parking time of less than 37 minutes.

The parking lot manager aims to have 95 % of the cars park for between 10 and 50 minutes. Is this goal being met?

E.5486 Consider a random variable $\mathcal{X}$ following a normal distribution with parameter 3 and σ^2.

Determine a value of σ rounded to the nearest thousandth so that the random variable $\mathcal{X}$ verifies the following probabilities:

a. $\mathcal{P}(\mathcal{X}\leqslant 2) = 0.4$ b. $\mathcal{P}(|\mathcal{X}-3|\leqslant 1) = 0.45$

E.6328 On a bottling line at a brewery, the quantity x *(in $c\ell$)* of liquid supplied by the machine to fill each bottle of capacity $110\,c\ell$, can be modeled by a random variable $\mathcal{X}$ of normal distribution with mean μ and standard deviation $\sigma = 2$.

Legislation requires that there be less than 0.1 % bottles containing less than one liter.
To what average value μ, rounded to the nearest unit, must the machine be set to comply with this legislation?

E.6327 The lifetime of a certain type of device is modeled by a random variable following a normal distribution of unknown mean and standard deviation. The specifications imply that 80 % of device production has a lifetime between 120 and 200 days and that 5 % of production has a lifetime of less than 120 days.

1. What are the values of μ and σ^2? *(We'll round these values to the nearest tenth)*
2. What is the probability, rounded to the nearest thousandth, of having a device with a lifetime between 200 days and 230 days?

8. *Binomial law and Moivre-Laplace theorem*

E.5494 Consider a random variable $\mathcal{X}$ following a binomial distribution with parameters 75 and 0.4 *($\mathcal{X} \sim \mathcal{B}(75\,;0.4)$)*.

1. Determine the expected value, variance, and standard deviation of the random variable $\mathcal{X}$ *(values rounded to three decimal places)*.
2. Using a calculator, determine the probabilities rounded to three decimal places:
   a. $\mathcal{P}\big(18 \leqslant \mathcal{X} \leqslant 46\big)$ b. $\mathcal{P}\big(\mathcal{X} \leqslant 25\big)$ c. $\mathcal{P}\big(\mathcal{X} \geqslant 23\big)$

Reminders: We have the following two properties of the expectation and variance of a random variable $\mathcal{W}$:
$E(a \cdot \mathcal{W} + b) = a \cdot E(\mathcal{W}) + b \quad ; \quad V(a \cdot \mathcal{W} + b) = a^2 \cdot V(\mathcal{W})$
where a and b are real numbers.

Noting $\mathcal{Z}$ the random variable following the reduced centered normal distribution *($\mathcal{Z} \sim \mathcal{N}(0\,;1)$)*, we consider the random variable $\mathcal{Y}$ defined by the relation:
$\mathcal{Y} = \sigma(\mathcal{X}) \cdot \mathcal{Z} + E(\mathcal{X})$

3. a. Establish the following equalities:
   $E(\mathcal{Y}) = E(\mathcal{X}) \quad ; \quad V(\mathcal{Y}) = V(\mathcal{X})$
   b. Justify that the random variable $\mathcal{Y}$ follows a normal distribution with parameters 30 and 18.
   c. Give the probabilities rounded to three decimal places:
   $\mathcal{P}\big(18 \leqslant \mathcal{Y} \leqslant 46\big) \quad ; \quad \mathcal{P}\big(\mathcal{Y} \leqslant 25\big) \quad ; \quad \mathcal{P}\big(\mathcal{Y} \geqslant 23\big)$
4. a. For all real numbers a and b vérifiant $a < b$, make a conjecture as to the comparison of the two probabilities below:
   $\mathcal{P}\big(a \leqslant \mathcal{X} \leqslant b\big) \quad ; \quad \mathcal{P}\big(a \leqslant \mathcal{Y} \leqslant b\big)$
   b. Which theorem from the course justifies this conjecture?

E.5513 In a company, we are interested in the probability that an employee will be absent during a flu epidemic. This company employs 220 employees. For the purposes of this exercise, we assume that the probability of an employee being sick during a given week during this epidemic period is equal to $p = 0.05$.

We assume that an employee's state of health does not depend on the state of health of their colleagues.

We denote by $\mathcal{X}$ the random variable that gives the number of employees who are sick in a given week.

1. Justify that the random variable $\mathcal{X}$ follows a binomial distribution, the parameters of which will be given. Calculate the mathematical expectation μ and standard deviation σ of the random variable $\mathcal{X}$.

We assume that the distribution of the random variable $\dfrac{\mathcal{X} - \mu}{\sigma}$ can be approximated by the standard normal distribution, i.e., with parameters 0 and 1.

We denote $\mathcal{Z}$ a random variable following the normal distribution centered reduced. The following table gives the probabilities of the event $\big\{\mathcal{Z} < x\big\}$ for some values of the real number x.

x	-1.55	-1.24	-0.93	-0.62	$-0{,}31$	0.00
$\mathcal{P}(\mathcal{Z} < x)$	0.061	0.108	0.177	0.268	0.379	0,500

x	0.31	0.62	0.93	1,24	1.55
$\mathcal{P}(\mathcal{Z} < x)$	0.621	0.732	0.823	0.892	0.939

2. Using the proposed approximation, calculate an approximate value of 10^{-2} close to the probability of the event: "*the number of employees absent from the company during a given week is greater than or equal to 7 and less than or equal to 15*".

E.5522 A factory produces large quantities of steel washers for the construction industry. Their diameter is expressed in millimeters.

The washers are marketed in batches of 1 000.
A batch of 1 000 is taken at random from a factory warehouse. This sampling is treated as a random draw of 1 000 washers.

Consider the random variable $\mathcal{X}$ which, for any sampling of 1 000 washers, associates the number of non-conforming washers among these 1 000 washers.

The random variable $\mathcal{X}$ is assumed to follow the binomial distribution with parameters $n=1000$ and $p=0.02$.

1. Determine the expectation, variance and standard deviation of the random variable $\mathcal{X}$.
2. a. Justify the following approximation: $\mathcal{P}\big(0\leqslant\mathcal{X}\leqslant15\big) \approx \mathcal{P}\big(-4{,}518\leqslant\mathcal{Z}\leqslant-1.129\big)$ where $\mathcal{Z}$ follows a normal distribution with parameters 0 and 1.
   b. Calculate the probability that there are at most 15 nonconforming washers in the batch of 1 000 washers, rounded to the nearest hundredth.

E.6955 A company manufactures spherical wooden balls.

The logs are then passed through a machine that randomly and equiprobably dyes them white, black, blue, yellow or red. After mixing, the logs are packed in bags. The quantity produced is sufficiently large for the filling of a sachet to be assimilated to a successive draw of marbles from the daily production.

A consumer study shows that children are particularly attracted to black marbles.

1. In this question only, the bags all consist of 40 marbles.
   a. A bag of marbles is chosen at random. Determine the probability that the chosen bag contains exactly 10 black marbles. Round the result to 10^{-3}.
   b. In a bag of 40 marbles, we counted 12 black marbles. Does this finding allow us to question the setting of the machine that dyes the marbles?
2. If the company wants the probability of obtaining at least one black bead in a bag to be greater than or equal to 99 %, what minimum number of beads must each bag contain to achieve this objective?

E.5493 Consider a random variable $\mathcal{X}$ following a binomial distribution with parameters 42 and 0,3 $(\mathcal{X}\sim\mathcal{B}(42\,;0{,}3))$.

1. Determine the expected value, variance, and standard deviation of the random variable $\mathcal{X}$ to the nearest thousandth.
2. Using a calculator, determine the value of the following probabilities to three decimal places:
   a. $\mathcal{P}\big(\mathcal{X}=13\big)$ b. $\mathcal{P}\big(8\leqslant\mathcal{X}\leqslant19\big)$
3. a. For all real numbers a and b satisfying $a<b$, justify the following approximation:
   $$\mathcal{P}\Big(a<\frac{\mathcal{X}-E(\mathcal{X})}{\sigma(\mathcal{X})}<b\Big) \approx \mathcal{P}\big(a\leqslant\mathcal{Z}\leqslant b\big) \quad \text{where } \mathcal{Z}\sim\mathcal{N}(0\,;1)$$
   b. Establish the following two approximations:
   - $\mathcal{P}\big(8\leqslant\mathcal{X}\leqslant19\big) \approx \mathcal{P}\big(-1{,}549\leqslant\mathcal{Z}\leqslant2{,}155\big)$
   - $\mathcal{P}\big(7{,}5\leqslant\mathcal{X}\leqslant19{,}5\big) \approx \mathcal{P}\big(-1{,}717\leqslant\mathcal{Z}\leqslant2{,}323\big)$

   c. Which of the two previous approximations is closest to the exact result obtained in question 2 b?
4. - Justify that the following approximation does not make sense: $\mathcal{P}\big(13\leqslant\mathcal{X}\leqslant13\big) \approx \mathcal{P}\big(0{,}135\leqslant\mathcal{Z}\leqslant0{,}135\big)$
   - Justify the following approximation and then give its value to the nearest thousandth: $\mathcal{P}\big(12{,}5\leqslant\mathcal{X}\leqslant13{,}5\big) = \mathcal{P}\big(-0{,}033\leqslant\mathcal{Z}\leqslant0{,}303\big)$

Note: Using Moivre-Laplace allows us to approximate the variable $\mathcal{X}$ following a binomial distribution *(, which is discrete with integer values)* , by a variable $\mathcal{X}_c$ following a normal distribution *(, which is continuous)*.

The probability $\mathcal{P}\big(\mathcal{X}=13\big)$ cannot be approximated by the probability $\mathcal{P}\big(\mathcal{X}_c=13\big)$ because the variable $\mathcal{X}_c$ is continuous and:
$\mathcal{P}\big(\mathcal{X}_c=13\big) = 0$
Instead, we approximate it by $\mathcal{P}\big(12{,}5\leqslant\mathcal{X}_c\leqslant13{,}5\big)$

Similarly, we will establish the approximation:
$\mathcal{P}\big(8\leqslant\mathcal{X}\leqslant19\big) = \mathcal{P}\big(7{,}5\leqslant\mathcal{X}_c\leqslant19{,}5\big)$

This approximation principle is called "**continuity correction**"

9. *Normal law: frequency fluctuation interval*

E.5517 Consider a random variable $\mathcal{X}_n$ suivant a binomial distribution with parameter n and p $(\mathcal{X}_n\sim\mathcal{B}(n\,;p))$.

Consider the random variable $\mathcal{Z}_n$ defined by:
$$\mathcal{Z}_n=\frac{\mathcal{X}_n-n{\cdot}p}{\sqrt{n{\cdot}p{\cdot}(1-p)}}$$

1. Using the properties of the expectation and variance of a random variable:
   $E\big(a{\cdot}\mathcal{X}+b\big) = a{\cdot}E\big(\mathcal{X}\big) + b \quad ; \quad V\big(a{\cdot}\mathcal{X}+b\big) = a^2{\cdot}V\big(\mathcal{X}\big)$

   Determine the expectation and variance of the random variable $\mathcal{Z}_n$.
2. a. What can we say about the following limit value:
   $\lim\limits_{n\mapsto+\infty} \mathcal{P}\big(a\leqslant\mathcal{Z}_n\leqslant b\big)$

   b. Determine the value of the limit:
   $\lim\limits_{n\mapsto+\infty} \mathcal{P}\big(-1.96\leqslant\mathcal{Z}_n\leqslant1.96\big)$
3. We note $\mathcal{F}_n=\dfrac{\mathcal{X}_n}{n}$. Show that for an integer n assez grand, we have:
   $$\mathcal{P}\Big(p-1.96\frac{\sqrt{p(1-p)}}{\sqrt{n}}\leqslant\mathcal{F}_n\leqslant p+1.96\frac{\sqrt{p(1-p)}}{\sqrt{n}}\Big)\approx0.95$$

E.5518 A lottery offers 200 tickets of which 50 are winning tickets. Consider the variable $\mathcal{X}$ giving, out of a randomly chosen batch of 40 tickets, the number of winning tickets. It is assumed that the number of tickets is large enough to assimilate the drawing of 40 tickets to a discounted draw game *(drawing a ticket does not alter the probabilities of the game)*.

1. Specify the law followed by the random variable $\mathcal{X}$, its expectation and standard deviation.
2. Studying the binomial distribution, give the frequency fluctuation interval at the threshold of 95 %.
3. a. Determine the asymptotic fluctuation interval of the frequency at the threshold of 95 %. *(bounds are rounded to the nearest thousandth)*.
   b. A high roller buys 40 tickets, but only has 7 tickets winning a prize. Is his draw representative at the threshold of 95 % *(we'll use the asymptotic fluctuation interval at the threshold of 95 %)*.

E.6072 For advertising purposes, the wholesaler displays on its leaflets: "88 % *of our tea guaranteed pesticide-free*".

An inspector from the brigade de répression des fraudes wishes to investigate the validity of the assertion. To this end, he takes 50 boxes at random from the wholesaler's stock and finds 12 with traces of pesticides.

It is assumed that, in the wholesaler's stock, the proportion of boxes without pesticide traces is indeed equal to 0.88.
We denote $\mathcal{F}$ the random variable which, to any sample of 50 boxes, associates the frequency of boxes containing no traces of pesticides.

1. Give the asymptotic fluctuation interval of the random variable $\mathcal{F}$ at the threshold of 95 %. *(values approximated to the nearest thousandth will be used)*.
2. Can the enforcement inspector decide, at the threshold of 95 %, that the advertisement is misleading?

E.6074 In a factory, a machine is used to manufacture parts. The machine is considered to be properly set up if 90 % of the parts it manufactures conform.

We decide to check this machine by examining n parts chosen at random *(n natural number)* in the machine's production. We assimilate these n draws to successive independent, discounted draws.

We note $\mathcal{X}_n$ the number of pieces that conform in the sample of n pieces, and $F_n = \dfrac{\mathcal{X}_n}{n}$ the corresponding proportion.

1. Justify that the random variable $\mathcal{X}_n$ follows a binomial distribution and specify its parameters.
2. In this question, we take $n=150$.
   Determine the asymptotic fluctuation interval I at the threshold of 95 % of the variable random variable F_{150} *(we'll use values approximated to the thousandth)*.
3. A quality test counts 21 non-conforming parts out of a sample of 150 parts produced.
   Does this call into question the machine setting? Justify the answer.

10. *Normal law: hypothesis and fluctuation range*

E.5516 A coin is assumed to be balanced. We toss 100 times this coin and consider the random variable $\mathcal{X}$ counting the number of times the face "*pile*" has appeared.

1. Give the law of the random variable $\mathcal{X}$, its expectation, variance and standard deviation.
2. We define the random variable $\mathcal{F}$ defined by: $\mathcal{F}=\dfrac{\mathcal{X}}{100}$
   a. Determine the asymptotic fluctuation interval of the frequency at the threshold of 95 %.
   b. After 100 coin tosses, John gets 38 times the "*pile*" face.
   Can we reject, at the threshold of 5 %, the hypothesis that the coin is balanced?

E.6959 A bulb manufacturer seeks to improve its quality and claims that there are no more than 6 % defective bulbs in its production. A consumer association carries out a test on a sample and obtains 71 defective bulbs out of 1 000.

1. In the case of exactly 6 % defective bulbs, determine an asymptotic fluctuation interval at the threshold of 95 % of the frequency of defective bulbs on a random sample of size 1 000.
2. Is there reason to question the company's assertion?

E.6424 We know that 39 % of the French population has blood group $A+$.
We want to know if this proportion is the same among blood donors.
183 blood donors are questioned and among them, 34 % are blood group $A+$.

Indicate whether the following statement is true or false and justify the answer:

"*We cannot reject, at the threshold of 5 %, the hypothesis that the proportion of people of blood group A among blood donors is 39 % as in the general population.*"

E.6969 Software detects unwanted messages called spam *(malicious messages, advertisements, ...)* and moves them to a file called "*folder spam*". The manufacturer estimates that 2.7 % of the messages moved to the spam folder are reliable messages.

A company equipped with this software is testing its effectiveness. The secretariat takes the trouble to count the number of reliable messages among the messages moved. It finds 13 reliable messages among the 231 messages moved during one week.

Do these results call into question the manufacturer's claim?

11. *Normal law: confidence interval of the proportion*

E.6079 We want to find out how many students at a university know the meaning of the acronym URSSAF. To do this, we ask them to answer a multiple-choice questionnaire. Each student is asked to choose from three possible answers.

In a first survey, it is found that 240 students answer correctly, among the 400 questioned.

Give a confidence interval at the threshold of 95 % of the estimate of the proportion p of students knowing this acronym at this university.

E.5521 A company manufactures wooden parts in large series.

In this section, we consider a large quantity of parts to be delivered to a chain of hypermarkets. We consider a sample of 1000 pieces taken at random from this delivery.

The shipment is large enough to be considered as a random draw.
It can be seen that 961 pieces are flawless.

1. What is the frequency of parts without any defects among this delivery?

2. Let F be the random variable which, to any sample of 100 pieces taken at random and with delivery from this delivery, associates the frequency of pieces in this sample which are without defect.

   Determine the confidence interval at 95 % of the proportion p of defect-free parts in the large quantity to be delivered.

E.6956 An institute conducts a survey to find out the proportion of people in a given population who are in favor of a land development project. To do this, a random sample of people from this population is interviewed, and each person is asked a question.

It is assumed that n people have answered the question, and it is assumed that these people constitute a random sample of size n *(where n is a natural integer greater than* 50*)*.
Of these people, 29 % are in favor of the development project.

1. Give a confidence interval, at the confidence level of 95 %, of the proportion of people who are in favor of the project in the total population.

2. Determine the minimum value of the integer n for the confidence interval, at the confidence level of 95 %, to have a magnitude less than or equal to 0.04.

E.5523 A company mass-produces rectangular metal plates for the automotive industry.

Consider a large quantity of plates to be delivered to an electric vehicle assembly line. Consider a sample of 10 000 plates taken at random from this delivery. The delivery is large enough to be considered as a random draw.

It can be seen that 9394 plates are flawless.

1. In this sample, what is the proportion of defect-free plates?

2. Let F be the random variable which, to any sample of 10 000 pieces taken at random and with delivery from this delivery, associates the frequency of pieces in this sample which are without defect.

   Given the sample taken, determine a confidence interval at 95 % of defect-free parts contained in the large quantity to be delivered.

E.5519 *The three parts of this exercise can êbe treated independently. Approximate results should be rounded to* 10^{-2}.

Part A : Normal distribution

A bale of straw is randomly selected from the July 20 production 201.

To answer the following questions, we will use the table below où the variable $\mathcal{Z}$ is a centered and reduced normal random variable.

x	-3	-2	-1.242	-0.556	0.556	1.242	2	3
$\mathcal{P}(\mathcal{X}\leqslant x)$	0.001	0.023	0.107	0.289	0.711	0.893	0.977	0.999

1. Let $\mathcal{X}$ be the random variable that, for each bundle sampled, is its thickness, expressed in millimeters. We assume that $\mathcal{X}$ follows a normal distribution with mean 360 and standard deviation 18.
   Calculate the probability : $\mathcal{P}\big(350\leqslant\mathcal{X}\leqslant370\big)$.

2. Let $\mathcal{Y}$ be the random variable that, for each bundle sampled from that day's production, associates its density expressed in ${}^{kg}\!/_{m^3}$. We assume that $\mathcal{Y}$ follows a normal distribution with mean 100 and standard deviation 5.
   Calculate the probability that a bale taken from that day's production has a density between $90\,{}^{kg}\!/_{m^3}$ and $110\,{}^{kg}\!/_{m^3}$.

Part B : Binomial distribution

Consider a large stock of straw bales, part of which is intended for insulation. Let E denote the event : "

A bale taken at random from the stock complies with insulation standards".
We assume : $p(E)=0.4$.

5 bales of straw are randomly selected from the stock to verify compliance with insulation standards. The stock is large enough that this selection can be considered a draw with a return of 5 bales.

We consider the random variable $\mathcal{Z}$ which, for any sample of 5 bales thus defined, associates the number of bales of straw that comply with insulation standards.

1. Justify that the random variable $\mathcal{Z}$ follows a binomial distribution, specifying the parameters.
2. Calculate the probability that, in such a sample, all straw bales comply with insulation standards.
3. Calculate the probability that, in such a sample, at least four straw bales comply with insulation standards.

Part C : Confidence interval

In this part, we consider the straw bales produced on July 22 2011. We denote p by the unknown proportion of straw bales from this production that comply with insulation standards.

A random sample of 50 bales of straw is taken from this production. The production is large enough that this sampling can be considered a draw with replacement.
It is found that 37 bales of straw from this sample comply with insulation standards.

1. Determine the observed frequency of compliant bales in this sample.
2. Determine a confidence interval for the frequency p at a confidence level of 95 %.

E.7805 Let g be the function defined on the interval $[0\,;1]$ by:

$$g(x)=\frac{1}{1+x^2}$$

We denote $\mathscr{C}_g$ by its representative curve in an orthonormal coordinate system of the plane.

We denote: $J=\displaystyle\int_0^1 g(x)\,\mathrm{d}x$

The aim of this section is to evaluate the integral J using the probabilistic method described below.

We randomly select a point $M(x\,;y)$ by independently drawing its coordinates x and y at random according to the uniform distribution on $[0\,;1]$.
We assume that the probability p that a point drawn in this way lies under the curve $\mathscr{C}_g$ is equal to the integral J.

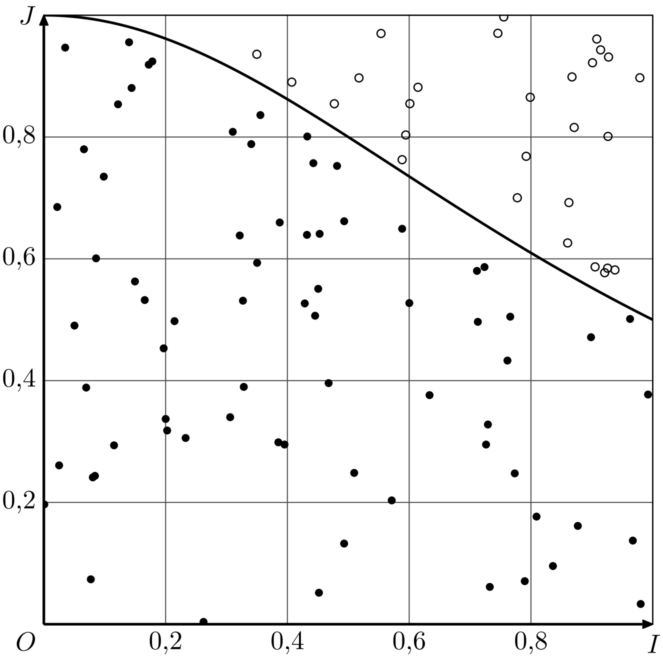
In practice, we initialize a counter c to 0, set a natural number n, and repeat the following process n times:

- two numbers x and y are chosen randomly and independently, according to the uniform distribution on $[0\,;1]$;
- if $M(x\,;y)$ is below the curve $\mathscr{C}_g$, increment the counter c by 1.

We assume that $f=\dfrac{c}{n}$ is an approximate value of J. *This is the principle of the Monte Carlo method.*

The figure below illustrates the method presented for $n=100$.

100 points have been placed randomly in the square.
The black discs correspond to the points below the curve, and the white discs correspond to the points above the curve.
The ratio of the number of black discs to the total number of discs gives an estimate of the area under the curve.



1. Copy and complete the algorithm below so that the function `estimate()` returns an approximate value of J.

```
def estimate(n)
    c ← ...
    For i ranging from 1 to ... do
       x ← random value between 0 and 1
       y ← ...
       Si ... then
           ... takes the value ...
        End If
    End For
    Return ...
```

The argument `n` specified when calling the function `estimation()` is the number of points chosen to perform this approximation.

2. For $n=1000$, the above algorithm gave the following result: $f=0.781$.
Give a confidence interval, at the confidence level of 95 %, for the exact value of J.
3. What must be the minimum value of n for the confidence interval, at a confidence level of 95 %, to have a range less than or equal to 0.02?

12. *Normal distribution and calculator*

E.7398 Consider the function f defined on $\mathbb{R}$ by the relation:

$$f(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{x^2}{2}}$$

Using the calculator, give the values of the integrals rounded to the nearest thousandth:

a. $\displaystyle\int_{-1}^{1} f(x)\,\mathrm{d}x=\ldots$ b. $\displaystyle\int_{-2}^{2} f(x)\,\mathrm{d}x=\ldots$ c. $\displaystyle\int_{-9}^{9} f(x)\,\mathrm{d}x=\ldots$

E.7399 Consider a random variable $\mathcal{X}$ following a probability law of mean 0 and standard deviation 1. Using a calculator, give the values of the following probabilities rounded to the thousandth:

a. $\mathcal{P}(\mathcal{X}\leqslant -1)$ b. $\mathcal{P}(\mathcal{X}\leqslant 2)$ c. $\mathcal{P}(\mathcal{X}\geqslant 1)$

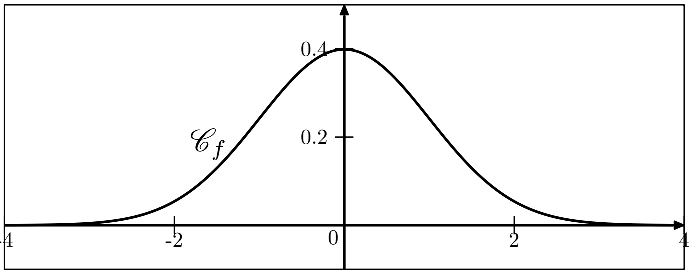
E.7400 Consider a random variable $\mathcal{X}$ following a probability distribution with mean 0 and standard deviation 1. Using a calculator, give the values of the following probabilities rounded to three decimal places:

a) $\mathcal{P}(-2\leqslant\mathcal{X}\leqslant3)$ b) $\mathcal{P}(0\leqslant\mathcal{X}\leqslant2)$ c) $\mathcal{P}(-4\leqslant\mathcal{X}\leqslant4)$

13. Normal distribution, properties and calculator

E.7401 Consider a random variable $\mathcal{X}$ following a normal distribution with mean 0 and standard deviation 1.

1. Give the value of: $\mathcal{P}(\mathcal{X}\leqslant0)$.
2. Note $\mathscr{C}_f$ the curve representing the density f of the random variable $\mathcal{X}$:



a) Interpret graphically the number $\mathcal{P}(\mathcal{X}\leqslant2)$.

b) Using a calculator, give the value of $\mathcal{P}(0\leqslant\mathcal{X}\leqslant2)$ rounded to the nearest thousandth.

c) Deduct the probability $\mathcal{P}(\mathcal{X}\leqslant2)$ rounded to the nearest thousandth.

E.7402 Consider a random variable $\mathcal{X}$ following a normal distribution with mean 0 and standard deviation 1. The probability rounded to three decimal places is given: $\mathcal{P}\left(\mathcal{X}\leqslant\frac{1}{2}\right)\approx0.691$

Without a calculator, determine the value of the following probabilities:

a) $\mathcal{P}\left(0\leqslant\mathcal{X}\leqslant\frac{1}{2}\right)$ b) $\mathcal{P}\left(-\frac{1}{2}\leqslant\mathcal{X}\leqslant0\right)$ c) $\mathcal{P}\left(\mathcal{X}\leqslant-\frac{1}{2}\right)$

14. Normal law and calculator

E.7403 Consider a random variable $\mathcal{X}$ following the normal distribution with mean 10 and standard deviation 3 .

Using a calculator, determine the following probabilities, rounded to the nearest thousandth:

a) $\mathcal{P}(\mathcal{X}\leqslant9)$ b) $\mathcal{P}(0\leqslant\mathcal{X}\leqslant12)$

E.7438 Let $\mathcal{X}$ be a random variable following a normal distribution with mean $\mu=3$ and standard deviation $\sigma=1$. Determine the following probabilities rounded to three decimal places:

a) $\mathcal{P}(\mathcal{X}\leqslant2)$ b) $\mathcal{P}(\mathcal{X}\geqslant3.5)$ c) $\mathcal{P}(2.5\leqslant\mathcal{X}\leqslant4)$

E.7439 A case is considered compliant if its thickness is between $19.8\,mm$ and $20.2\,mm$.
The supplier B wants at least 95 % of the cases produced to conform. To do this, he wants to check the settings of the production machines.
A case is chosen at random from the supplier's production B. Let $\mathcal{X}$ be the random variable associated with the thickness *(in mm)* of the case. It is assumed that $\mathcal{X}$ follows a normal distribution with expectation $20\,mm$.

Observing the settings of the production machines, the supplier B finds that the standard deviation of $\mathcal{X}$ is equal to 0.2. Justify that machine settings need to be reviewed.

E.7845 In January 2015, the director of a contemporary art museum commissioned a survey concerning visitor habits.

To manage visitor flows, part of the survey focused on the duration of a visit to this museum. It was established that the duration D of a visit, in minutes, follows the normal distribution of mean $\mu=90$ and standard deviation $\sigma=15$.

1. Determine $\mathcal{P}(90\leqslant D\leqslant120)$ then interpret this result in the context of the exercise.
2. The director states that he will increase the capacity of the museum's catering area if more than 2 % visitors stay more than 2 hours and 30 minutes per visit. What will his decision be then?

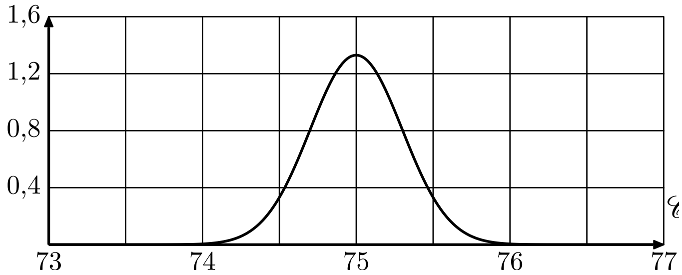
15. Normal distribution and property of the mean

E.7442 *Results will be rounded, if necessary, to* 10^{-3}.

A company manufactures circular medals in large quantities.

The diameter, expressed in millimeters, of a medal manufactured by this company conforms when it belongs to the interval $[74.4\,;75.6]$.

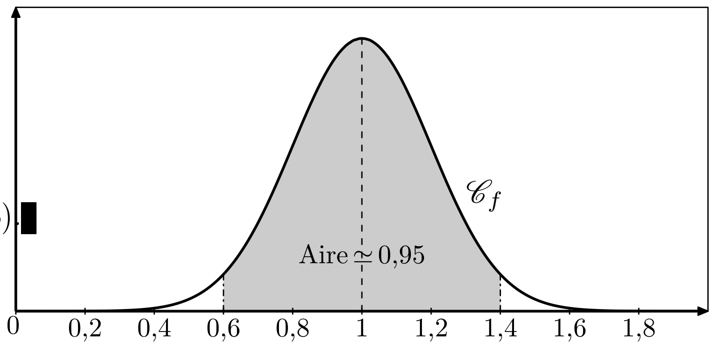
Note $\mathcal{Y}$ the random variable which, to each medal taken at random from the production, associates its diameter in millimetres. It is assumed that the random variable $\mathcal{Y}$ follows a normal distribution with mean μ and standard deviation 0.25.
The curve below is the graphical representation of $\mathcal{Y}$'s probability density.



① Indicate by graphical reading the value of μ.

② Using the calculator, determine the probability $\mathcal{P}(74.4 \leqslant \mathcal{Y} \leqslant 75.6)$.

E.7393 ⚠ Consider a random variable $\mathcal{X}$ following a normal distribution. The curve in the figure below represents the density function f associated with the variable $\mathcal{X}$:



Of the four proposals below, only one is correct. Which one?

ⓐ The expectation of $\mathcal{X}$ is 0.4.

ⓑ The expectation of $\mathcal{X}$ is 0.95.

ⓒ The standard deviation of $\mathcal{X}$ is approximately 0.4.

ⓓ The standard deviation of $\mathcal{X}$ is approximately 0.2.

(You can use the calculator to justify your choice.)

16. *Normal law and symmetry property*

E.7404 Consider a random variable $\mathcal{X}$ following a normal distribution with mean 3 and standard deviation 2. We give the probability rounded to the nearest thousandth :
$\mathcal{P}(\mathcal{X} \leqslant 4) \approx 0.691$

Without the aid of a calculator, determine the following probabilities :

ⓐ $\mathcal{P}(\mathcal{X} \leqslant 3)$ ⓑ $\mathcal{P}(3 \leqslant \mathcal{X} \leqslant 4)$

ⓒ $\mathcal{P}(2 \leqslant \mathcal{X} \leqslant 3)$ ⓓ $\mathcal{P}(\mathcal{X} \leqslant 2)$

17. *Normal distribution and graphical comparison*

E.7488 ⚠ A study is being conducted by an association that fights road violence. Observers on a boulevard in a large city studied the behavior of car drivers when passing through a traffic light.

Let $\mathcal{X}$ be the random variable that counts the number of cars per hour near a traffic light.
We assume that $\mathcal{X}$ follows a normal distribution with mean 3 000 and standard deviation 150.

Runde the values to the nearest thousandth.

① Using a calculator, determine the probability of counting between 2 800 and 3 200 cars per hour at this location.

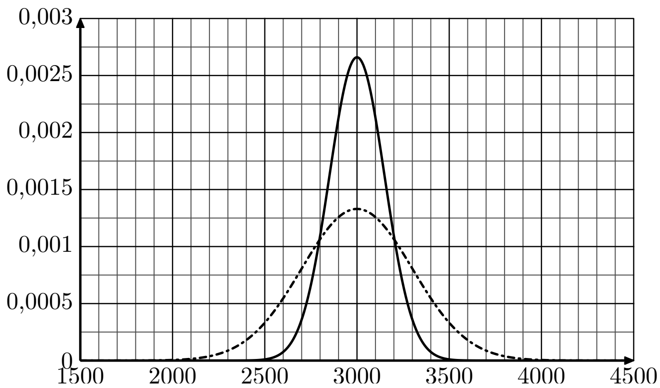
② Using a calculator, determine the probability of counting more than 3 100 cars per hour at this location.

③ *In this question, any evidence of research, even if incomplete, or initiative, even if unsuccessful, will be taken into account in the evaluation.*

At another location on the boulevard, near a bridge, the random variable $\mathcal{Y}$ that counts the number of cars per hour follows a normal distribution with a mean of 300 and a standard deviation of σ strictly greater than 150.

In the graph below, the curve corresponding to $\mathcal{X}$ is solid, and the curve corresponding to $\mathcal{Y}$ is dotted

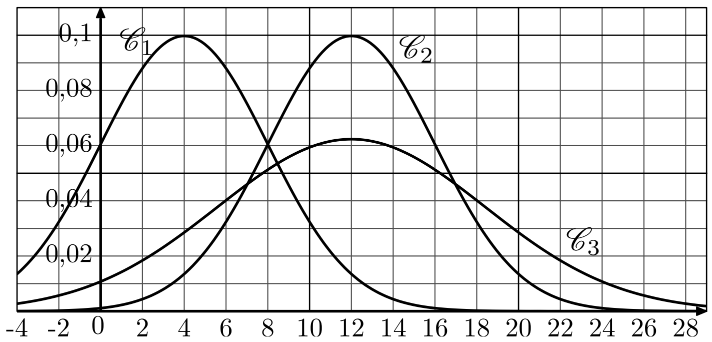
Determine where on the boulevard, near the traffic light or the bridge, the probability of 2 800 to 3 200 cars passing in one hour is highest. Justify your answer using the graph.



E.7391 The AFDIAG carried out a survey and found that celiac disease was diagnosed on average 11 years after the first symptoms.

We denote $\mathcal{X}$ the random variable representing the time in years taken to diagnose celiac disease from the onset of the first symptoms.

The distribution of $\mathcal{X}$ can be assimilated to the normal distribution with expectation $\mu=12$ and standard deviation $\sigma=4$.



1. Justify that the curve $\mathscr{C}_1$ is not the density curve of the random variable $\mathcal{X}$.
2. a. Using the calculator, determine the probability $\mathcal{P}(10\leqslant\mathcal{X}\leqslant12)$.

   b. Of the two curves $\mathscr{C}_2$ and $\mathscr{C}_3$, which is the representative curve of the density of $\mathcal{X}$.

18. Normal distribution, standard deviation and centered interval

E.7843 A company specializing in the customization of smartphone cases buys from a supplier.

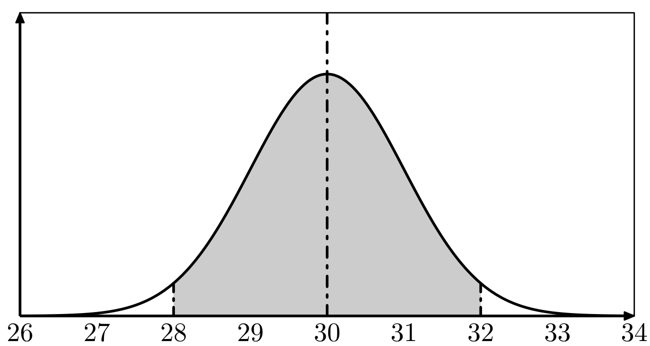
A case is considered compliant if its thickness is between $19.8\,mm$ and $20.2\,mm$.
The supplier wants at least 95 % of the cases produced to be compliant. To achieve this, he wants to check the settings of the production machines.
A case is chosen at random from the supplier's production.
Let $\mathcal{X}$ be the random variable associated with the thickness *(in mm)* of the case. It is assumed that $\mathcal{X}$ follows a normal distribution with expectation $20\,mm$.

1. Observing the settings of the production machines, the supplier B finds that the standard deviation of $\mathcal{X}$ is equal to 0.2.
   Justify that machine settings need to be reviewed.
2. Determine a value for the standard deviation of $\mathcal{X}$ for which the probability of a case conforming is approximately equal to 0.95.

E.7491 A key is said to conform for reading when its reading speed, expressed in $^{Mo}\!/_s$, belongs to the interval $[98\,;103]$. A key is said to be write compliant when its write speed expressed in $^{Mo}\!/_s$ belongs to the interval $[28\,;33]$.

1. We denote $\mathcal{R}$ the random variable which, to each key taken at random from the stock, associates its reading speed. It is assumed that the random variable R follows the normal distribution with expectation $\mu=100$ and standard deviation $\sigma=1$.
   Calculate the probability that a key is compliant for reading.
2. Note $\mathcal{W}$ the random variable which, each key taken at random from the stock, associates its writing speed. The random variable $\mathcal{W}$ is assumed to follow a normal distribution.

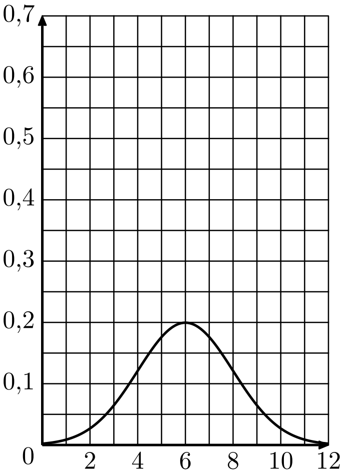
   The graph below represents the probability density of the random variable $\mathcal{W}$.



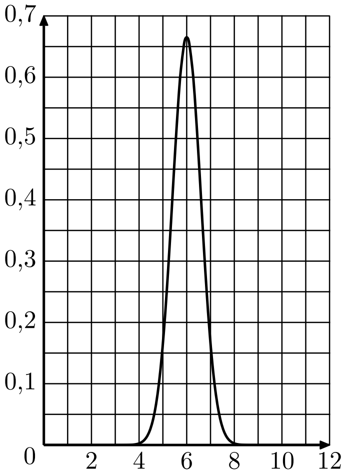
   The area unit is chosen so that the area under the curve is equal to one and the shaded area is approximately equal to 0.95 area unit. The straight line of equation $x=30$ is an axis of symmetry of the curve.

   Determine the expectation and standard deviation of the random variable $\mathcal{W}$. Justify

E.7394 The weekly training time of runners on the red course, expressed in hours, can be modeled by a random variable $\mathcal{X}$ that follows the normal distribution whose expectation is 6 hours and standard deviation is 2 hours.



Graphique 1



Graphique 3

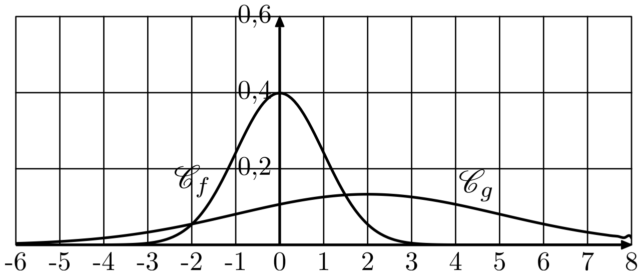
Which of the following two graphs represents the density function of the normal distribution with parameters $\mu=6$ and $\sigma=2$? Justify the answer.

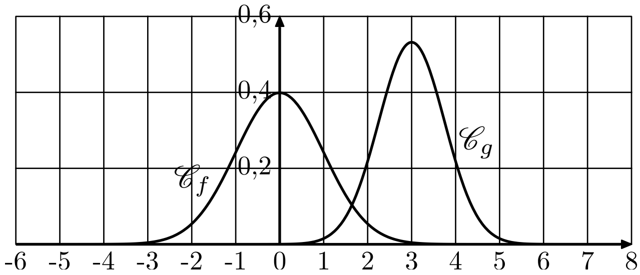
E.7437

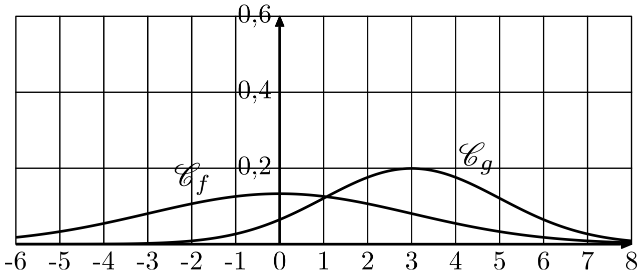
For the proposition below, only one of the four proposed answers is correct. Which one? Justify your answer.

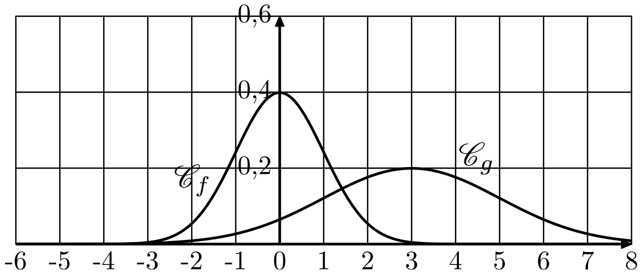
The function f is the probability density function associated with the reduced centered normal distribution $\mathcal{N}(0\,;\,1)$. The function g is the probability density function associated with the normal distribution with mean $\mu=3$ and standard deviation $\sigma=2$.

The graphical representation of these two functions is:

a 

b 

c 

d 

19. *Normal law and conditional probability*

E.7489 A customer in the store is concerned about the lifespan of the phone he has just bought of type T_1.
We note $\mathcal{X}$ the random variable which, to each cell phone of type T_1 taken at random from the production, associates its lifetime, in months.
The random variable $\mathcal{X}$ is assumed to follow the normal distribution with expectation $\mu=48$ and standard deviation $\sigma=10$.

1. Justify that the probability that the type T_1 phone sampled will operate for more than 3 years, i.e. 36 months, is approximately 0.885.
2. The sampled T_1 phone is known to have worked for more than 3 years. What is the probability that it will operate less than 5 years?

20. *Inverse normal distribution*

E.7834 A marathon is a running sporting event.
Assume that the time in minutes taken by a marathoner to finish the Tartonville Marathon is modeled by a random variable $\mathcal{T}$ which follows a normal distribution with expectation $\mu=250$ and standard deviation $\sigma=39$.

- Calculate: $\mathcal{P}(\mathcal{T}\leqslant 300)$
- By the method of your choice, estimate the value of the real number t, rounded to unity, verifying: $\mathcal{P}(\mathcal{T}\geqslant t)=0.9$.
- Interpret the result obtained in the exercise.

E.7167 Electric boats are equipped with a battery with an average operating time of 500 minutes.
Boat batteries are recharged only at the end of each day of use.
We denote $\mathcal{X}$ the random variable corresponding to the running time of a boat's battery, expressed in minutes. It is assumed that $\mathcal{X}$ follows the normal distribution with expectation $\mu=500$ and standard deviation $\sigma=10$.

We wish to know the duration of use, in minutes, per day of each boat so that only 1 % of the boats are unloaded before the end of the day.

E.7443

Note : This exercise is a multiple-choice quiz *(MCQ)*. For each question, only one of the four answers is correct. Copy down the question number and the correct answer. No justification is required. A correct answer is worth 1 points, while an incorrect answer or no answer is worth neither points nor penalties.

Researchers have designed a test to assess the reading speed of third-grade students. This test consists of timing the reading of a list of 20 words. This test was given to a large number of third-grade students. Let $\mathcal{X}$ be the random variable that gives the time in seconds taken by a third-grade student to complete the test. We assume that $\mathcal{X}$ follows a normal distribution with mean $\mu=32$ and standard deviation $\sigma=13$.

1. The probability $\mathcal{P}\big(19\leqslant\mathcal{X}\leqslant45\big)$ rounded to two decimal places is :

   a 0,50 b 0,68 c 0,84 d 0,95

2. Let t be the reading time satisfying $\mathcal{P}\big(\mathcal{X}\leqslant t\big)=0{,}9$. The value of t rounded to the nearest integer is :

   a $t=32s$ b $t=45s$ c $t=49s$ d $t=58s$

21. *Normal law : annales*

E.6985 A water sports center offers rentals of various boats for visiting the Verdon Gorge. Tourists can rent kayaks, pedal boats, or electric boats for 1 hours or 2 hours.

The parts **A** *et* **B** *are independent*

Part A

A statistical study shows that :

- 40 % of the boats rented are pedal boats ;
- 35 % of the boats rented are kayaks ;
- The other boats rented are electric boats ;
- 60 % of the pedal boats are rented for a period of 1 hours ;
- 70 % of the kayaks are rented for a period of 1 hours ;
- half of the electric boats are rented for a period of 1 hours.

A tourist who comes to rent a boat is interviewed at random. The following events are noted : A, B, C, D, and E :

- A : "*the rented boat is a pedal boat*" ;
- B : "*the rented boat is a kayak*" ;
- C : "*the rented boat is an electric boat*" ;
- D : "*the boat is rented for a period of* 1 *hours*" ;
- E : "*the boat is rented for a period of* 2 *hours*".

1. Translate the situation into a weighted tree
2. Calculate the probability that the boat is rented for a period of 2 hours is equal to 0.39.
3. Knowing that the boat was rented for 2 hours, what is the probability that it is an electric boat? Round the result to two decimal places.
4. The nautical base charges the following rates :

	1 hour	2 hours
Pedal boat	15 €	25 €
Kayak	10 €	16 €
Electric boat	35 €	60 €

   On average, 200 boats are rented per day. Determine the daily revenue that the nautical base can expect to earn

Part B

In this part, the results will be rounded to the nearest thousandth

Electric boats are equipped with a battery with an average autonomy of 500 minutes
The boat batteries are only recharged at the end of each day of use.
Let $\mathcal{X}$ be the random variable corresponding to the operating time of a boat's battery, expressed in minutes. We assume that $\mathcal{X}$ follows a normal distribution with mean $\mu=500$ and standard deviation $\sigma=10$.

1. Using the calculator, calculate $\mathcal{P}\big(490<\mathcal{X}<520\big)$.
2. Each day, the boats are used for 8 hours without being recharged.
   Determine the probability that a boat's battery will be discharged before the end of the day.
3. Determine the integer a such that : $\mathcal{P}\big(\mathcal{X}<a\big)\approx0.01$
   Interpret the result in the context of the exercise.

E.6991

The three parts of this exercise are independent.

Part A

According to the "*report on driver's license exams*" for the year 2014 published by the Department of the Interior in November 2015, 20 % of the people who took the practical driving test had completed the early driver training program *(AAC)*.
Of these candidates, 75 % passed the test. For candidates who did not follow the AAC program, the pass rate for the test was only 56.6 %.
One of the candidates for the practical driving test is chosen at random in 2014.
Consider the following events:

- A "*the candidate took the AAC course*";
- R "*the candidate passed the exam*".

Remember that if E and F are two events, the probability of the opposite event of E.

1. a. Give the probabilities $\mathcal{P}(A)$, $\mathcal{P}_A(R)$, and $\mathcal{P}_{\overline{A}}(R)$.
   b. Translate the situation into a weighted tree.
2. a. Calculate the probability $\mathcal{P}(A\cap R)$.
   b. Interpret this result in the context of the statement.
3. Justify that: $\mathcal{P}(R)=0.6028$
4. Given that the candidate passed the exam, calculate the probability that they followed the AAC course, then give the value rounded to the nearest 10^{-4}

Part B

A driving school manager claims that for the year 2016, the probability of passing the exam is equal to 0.62.
Having doubts about this statement, a motorists' association decides to survey 400 exam candidates out of 2016. It turns out that 220 of them did indeed obtain their driver's license.

1. Determine an asymptotic fluctuation interval at the threshold of 95 % for the frequency of candidates who passed in a random sample of 400 candidates.
2. Can we question the statement made by the manager of this driving school?
   Justify your answer.

Part C

According to a survey conducted in 2013 by the association "*Road Safety*", , the average cost of obtaining a driver's license was approximately 1 500 €. We decide to model the cost of obtaining a driver's license using a random variable $\mathcal{X}$ that follows a normal distribution with mean $\mu=1\,500$ and standard deviation $\sigma=410$

1. Determine the probability, rounded to the nearest 10^{-2}, that the cost of a driver's license is between 1 090 € and 1 910 €.
2. Determine: $\mathcal{P}(\mathcal{X}\leqslant 1\,155)$
   Give the result rounded to 10^{-2}.
3. a. Using the method of your choice, estimate the value of the real number a rounded to the nearest whole number, checking:
   $\mathcal{P}(\mathcal{X}\geqslant a)=0.2$.

b. Interpret this result in the context of the statement.

E.6972

A marathon is a competitive running event.
In this exercise, all results will be rounded to the nearest 10^{-3}.
The two parts of this exercise are independent.

Part A

A study of the Tartonville marathon shows that:

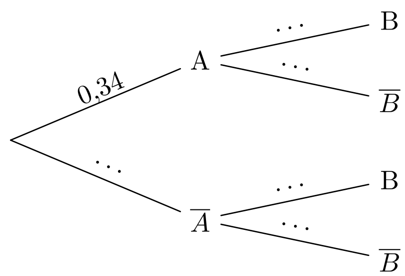
- 34 % of runners finish the race in less than 234 minutes;
- among runners who finish the race in less than 234 minutes, 5 % are over 60 years old;
- among runners who finish the race in more than 234 minutes, 84 % are under 60 years old.

A runner is selected at random and the following events are considered:

- A: "*the runner finished the marathon in less than* 234 *minutes*";
- B: "*the runner is under* 60 *years old*".

Remember that if E and F are two events, the probability of event E is denoted by $\mathcal{P}(E)$ and that of E given F is denoted by $\mathcal{P}_F(E)$. Furthermore, $\overline{E}$ denotes the opposite event of E.

1. Copy and complete the probability tree below associated with the situation in the exercise:



2. a. Calculate the probability that the person chosen finished the marathon in less than 234 minutes and is âover 60 years old
   b. Check that: $\mathcal{P}(\overline{B})=0.123$
   c. Calculate $\mathcal{P}_{\overline{B}}(A)$ and interpret the result in the context of the exercise.

Part B

Suppose that the time in minutes taken by a marathon runner to finish the Tartonville marathon is modeled by a random variable $\mathcal{T}$ that follows a normal distribution with mean $\mu=250$ and standard deviation $\sigma=39$.

1. Calculate $\mathcal{P}(210\leqslant\mathcal{T}\leqslant 270)$
2. A runner is chosen at random from among the runners who took between 210 minutes and 270 minutes to finish the marathon.

   Calculate the probability that this runner finished the race in less than 240 minutes.
3. a. Calculate $\mathcal{P}(\mathcal{T}\leqslant 300)$
   b. Using the method of your choice, estimate the value of the real number t, rounded to the nearest whole number, checking:
   $\mathcal{P}(\mathcal{T}\geqslant t)=0.9$
   c. Interpret the result obtained in the exercise.

E.6977 According to AFDIAG *(French Association for Gluten Intolerance)*, celiac disease, also known as gluten intolerance, is one of the most common digestive disorders. It affects approximately 1 % of the population.

It is estimated that only 20 % of people with gluten intolerance are tested and diagnosed.

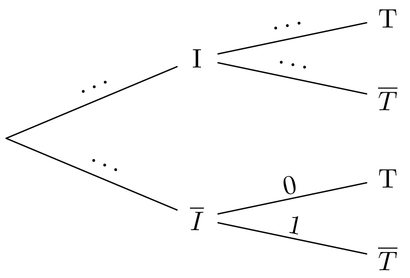
It is assumed that if a person is not gluten intolerant, they will not take the test to be diagnosed.

A person is chosen at random from the French population, which numbered approximately 66.6 million inhabitants on January 1er, 2016.
The following events are considered:

- I: "*the person chosen is gluten intolerant*";
- T: "*the chosen person takes the test to be diagnosed*"

Part A

1. Copy and complete the probability tree below:

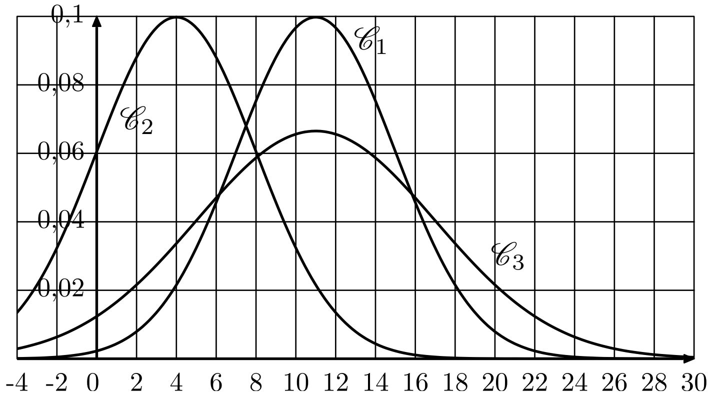


2. Calculate the probability that the chosen person is gluten intolerant and does not pass the test to be diagnosed.
3. Show that: $\mathcal{P}(T) = 0.002$

Part B

The AFDIAG conducted a survey and found that celiac disease was diagnosed on average 11 years after the first symptoms appeared.
Let $\mathcal{X}$ be the random variable representing the time in years taken to diagnose cœliac disease from the onset of the first symptoms.
It is assumed that the law of $\mathcal{X}$ can be assimilated to the normal law of expectation $\mu = 11$ and standard deviation $\sigma = 4$.

1. Calculer the probability of the disease being diagnosed between 9 years and 13 years after the first symptoms. Round the result to 10^{-3}.
2. Calculate $\mathcal{P}(\mathcal{X} \leqslant 6)$. Round the result to 10^{-3}.
3. Knowing that $\mathcal{P}(\mathcal{X} \leqslant a) = 0.84$, give the value of a rounded to unity.
   Interpret the result in the context of the exercise.
4. Which of these three curves represents the density function of the normal distribution of expectation $\mu = 11$ and standard deviation $\sigma = 4$? Justify your choice. Use the answers to the previous questions as a guide.



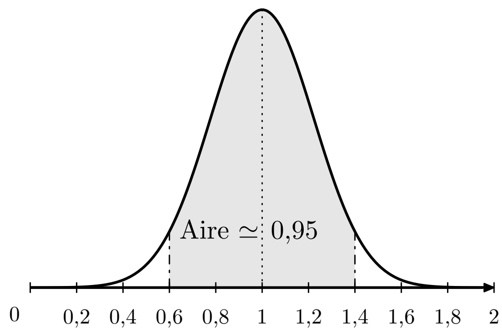
E.6979

This exercise is a multiple-choice questionnaire. For each of the following questions, only one of the four answers provided is correct. No justification is required.
A correct answer earns one point. A multiple incorrect answer or no answer does not earn or deduct any points.
Indicate the question number and the corresponding answer on your answer sheet.

1. Consider the function g defined on $]0\,;+\infty[$ by:
   $$g(x) = \frac{2}{x}$$
   The average value of the function g over the interval $[1\,;\mathrm{e}]$ is:

   a) 2 b) $\dfrac{1}{\mathrm{e}-1}$ c) $\dfrac{2}{\mathrm{e}-1}$ d) $\dfrac{-2}{\mathrm{e}-1}$

2. Consider a random variable $\mathcal{X}$ following a normal distribution. The curve below represents the density function f associated with the variable $\mathcal{X}$.



   a) The expectation of $\mathcal{X}$ is 0.4.
   b) The expectation of $\mathcal{X}$ is 0.95.
   c) The standard deviation of $\mathcal{X}$ is approximately 0.4.
   d) The standard deviation of $\mathcal{X}$ is approximately 0.2.

3. To celebrate its opening, a hypermarket is offering its customers a scratch card for every 10 euros spent. *The hypermarket* ***claims*** *that 15 % of the scratch cards are winners*, , meaning they will entitle the holder to a 5 euros voucher.

   Amandine received 50 scratch cards after spending 500 euros in this hypermarket. Two of them were winners.

   We assume that the number of scratch cards is large enough to consider that a sample of 50 cards corresponds to a random draw with replacement.

   a) The asymptotic fluctuation interval at the threshold of 95 % of the observed frequency of winning tickets in a sample of 50 scratch cards is $[0.051\,;0.249]$, with the limits rounded to the nearest thousandth.

(b) The asymptotic fluctuation interval at the threshold of 95 % of the observed frequency of winning tickets in a sample of 50 scratch cards is $[0.100\,;0.200]$, with the limits rounded to the nearest thousandth.

(c) The frequency of winning tickets received by Amandine is $\dfrac{50}{500}$.

(d) Amandine can announce with a risk of 5 % that the hypermarket's claim is not false.

E.6986

This exercise is a multiple-choice questionnaire. For each of the following questions, only one of the four answers provided is correct. No justification is required. A correct answer earns one point. An incorrect answer, multiple answers, or no answer to a question do not earn or deduct points.

Indicate the question number and the corresponding answer on your answer sheet.

(1) A and B are two events in a random experiment. We denote the opposite event of B by $\overline{B}$. We know that:
$\mathcal{P}(A)=0.6 \quad ; \quad \mathcal{P}(B)=0.5 \quad ; \quad \mathcal{P}(A\cap B)=0.42$
We can say that:

(a) $\mathcal{P}_A(B)=0.3$ (b) $\mathcal{P}(A\cup B)=0.58$

(c) $\mathcal{P}_B(A)=0.84$ (d) $\mathcal{P}(A\cap\overline{B})=0.28$

(2) At a ski resort, the waiting time at a given chairlift, expressed in minutes, can be modeled by a random variable $\mathcal{X}$ that follows a uniform distribution on the interval $[0\,;5]$.

(a) The expectation of this distribution $\mathcal{X}$ is $\dfrac{2}{5}$

(b) $\mathcal{P}(\mathcal{X}>2)=\dfrac{3}{5}$ (c) $\mathcal{P}(\mathcal{X}\leqslant 2)=\dfrac{3}{5}$

(d) $\mathcal{P}(\mathcal{X}\leqslant 5)=0$

(3) A machine fills bottles with a stated volume of $100\,m\ell$. We assume that the volume contained in the bottle can be modeled by a random variable $\mathcal{Y}$ that follows a normal distribution with expected value $100\,m\ell$ and standard deviation $2\,m\ell$.

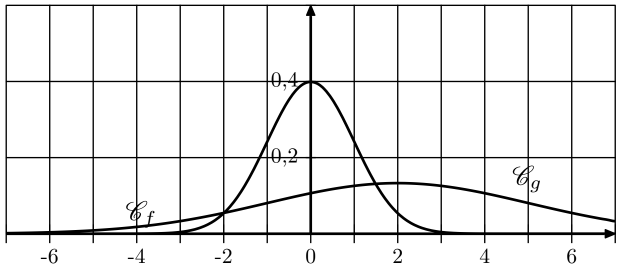
(a) $\mathcal{P}(\mathcal{Y}\leqslant 100)=0.45$ (b) $\mathcal{P}(\mathcal{Y}>98)=0.75$

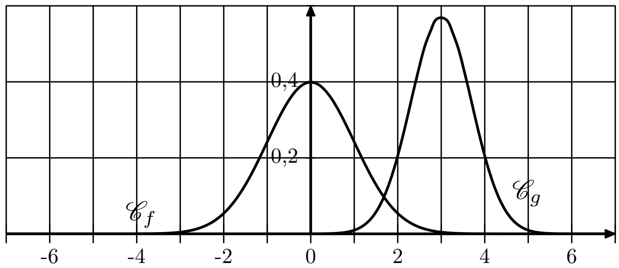
(c) $\mathcal{P}(96\leqslant\mathcal{Y}\leqslant 104)\approx 0.95$ (d) $\mathcal{P}(\mathcal{Y}\leqslant 110)\approx 0.85$

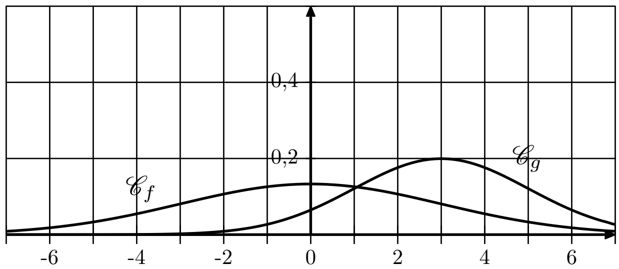
(4) A newspaper article claims that in France, there are 16 % left-handed people. A researcher wants to verify this claim. To do this, he wants to determine the sample size of the French population to be studied that would yield a confidence interval with a width equal to 0.1 at a confidence level of 0.95. The sample size is:

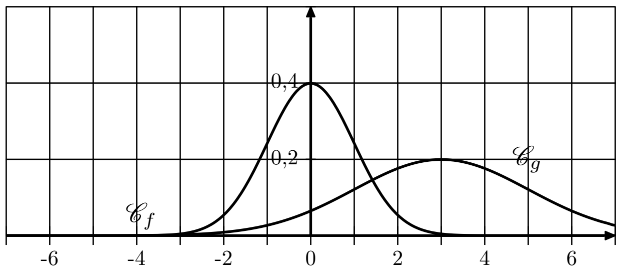
(a) 30 (b) 64 (c) 100 (d) 400

(5) The function f is the probability density function associated with the reduced centered normal distribution $\mathcal{N}(0\,;1)$.
The function g is the probability density function associated with the normal distribution with mean $\mu=3$ and standard deviation $\sigma=2$. The graphical representation of these functions is:

(a)


(b)


(c)


(d)


22. *Asymptotic fluctuation interval*

E.7842 A company specializing in the personalization of smartphone cases buys from a supplier who guarantees 94 % non-defective cases.

An employee of the company takes a sample of 400 cases that come from his supplier.
He finds that 350 of these cases are not defective.

(1) Determine an asymptotic fluctuation interval at the threshold of 95 % of the frequency of defective cases in a random sample of 400 cases from the supplier.
Approximate values to the thousandth of the bounds of this interval will be given.

(2) Should the supplier be informed of a problem?

E.7836 *This exercise is a multiple-choice questionnaire. Of the four answers provided, only one is correct. Which one? Justify your answer.*

To celebrate its opening, a hypermarket is offering its customers a scratch card for every 10 euros spent. The hypermarket claims that 15 % of the scratch cards are winners, meaning they will entitle the holder to a 5 euro voucher.

Amandine received 50 scratch cards after spending 500 euros in this hypermarket. Two of them were winners.

We assume that the number of scratch cards is large enough to consider that a sample of 50 cards corresponds to a random draw with replacement.

(a) The asymptotic fluctuation interval at the 95 % threshold of the observed frequency of winning tickets in a sample of 50 scratch cards is $[0.051\,;0.249]$, with the limits rounded to the nearest thousandth.

(b) The asymptotic fluctuation interval at the 95 % threshold of the observed frequency of winning tickets in a sample of 50 scratch cards is $[0.100\,;0.200]$, with the limits rounded to the nearest thousandth.

(c) The frequency of winning tickets received by Amandine is $\dfrac{50}{500}$.

(d) Amandine can announce with a risk of 5 % that the hypermarket's claim is not false.

E.7839 A driving school manager asserts that for the year 2016, the probability of passing the exam is equal to 0.62.
Having doubts about this claim, a motorists' association decides to question 400 exam candidates from among those of 2016. It turns out that 220 of them actually got the driving license.

(1) Determine an asymptotic fluctuation interval at the threshold of 95 % of the frequency of successful candidates in a random sample of 400 candidates.

(2) Can we cast doubt on the claim made by the manager of this driving school?
Justify your answer.

E.7840 *Results will be rounded to the nearest thousandth.*

On the front of his store, the supermarket manager displays:

"*More than 90 % of our store's customers are satisfied with the introduction of our automatic checkouts*"

A consumer association wishes to examine this claim. To do so, it carries out a survey: 860 customers are questioned, and 763 of them say they are satisfied with the introduction of these automatic checkouts.

Does this call into question the manager's assertion?

E.7844 In January 2015, the director of a contemporary art museum commissions a survey concerning visitor habits.

Of all contemporary art museums, 22 % of visitors are of foreign nationality. Out of a random sample of 2 000 visitors to the museum previously considered, 490 visitors are of foreign nationality.
What can the director of this museum conclude from this? Please explain.

23. *Confidence interval*

E.7835 *Of the answers given, only one is correct. Which one? Justify your answer.*

A company manufactures metal tubes of width $2\,m$.
A metal tube is considered to be within the standard if its length is between $1.98\,m$ and $2.02\,m$. A sample of 1 000 tubes is taken at random, and it is observed that 954 tubes are within the norm.

The confidence interval for the frequency of tubes in the standard for this company at the confidence level of 95 %, with bounds rounded to 10^{-3} is:

(a) $[0.922\,;\,0.986]$ (b) $[0.947\,;\,0.961]$

(c) $[1.98\,;\,2.02]$ (d) $[0.953\,;\,0.955]$

E.7841 *Of the answers given, only one is correct. Which one? Justify your answer.*

A polling institute is carrying out a survey to measure the degree of satisfaction with a company's after-sales service. An initial survey of a random sample of 500 customers reveals that there are 438 satisfied customers. A confidence interval at the 0.95 confidence level for estimating the proportion of satisfied customers is:

(a) $[0.079\,;\,0.169]$ (b) $[0.455\,;\,0.545]$

(c) $[0.831\,;\,0.921]$ (d) $[0.874\,;\,0.878]$

24. *Confidence interval and amplitude*

E.7833 Of the questions below, only one of the proposed answers is correct. Which one? Justify your answer.

The aim of a health survey is to estimate the proportion of people who comply with the vaccination schedule recommended by the Haut Conseil de la Santé Publique. To obtain a confidence interval of magnitude 0.01 at confidence level 0.95 of this proportion, we need to question:

(a) 200 people (b) 400 personnes

(c) 10 000 personnes (d) 40 000 personnes

E.7838 A newspaper article claims that 16 % are left-handed in France. A researcher wishes to verify this assertion. To do this, he wants to determine the sample size of the French population to be studied that would enable him to obtain a confidence interval of amplitude equal to 0.1 at a confidence level of 0.95. The sample size is:

(a) 30 (b) 64 (c) 100 (d) 400

E.7848 A laboratory wishes to test the efficacy of a drug. The minimum number of patients to question to obtain a confidence interval of length less than or equal to 0.01 is:

(a) 200 (b) 40 000 (c) 4 000 (d) 1 000

25. *Probability: continuous law*

E.5539 Consider the random variable $\mathcal{X}$ following a normal distribution with mean 54 and standard deviation 5 *($\mathcal{X}\sim\mathcal{N}(54\,;25)$)*.

We'll use the value table below where the random variable $\mathcal{Z}$ follows a centered and reduced normal distribution *($\mathcal{Z}\sim\mathcal{N}(0\,;1)$)*:

x	-2.8	-1.8	-1.2	0.4	1.2	2
$\mathcal{P}(\mathcal{Z}\leqslant x)$	0.003	0.004	0.115	0.655	0.885	0.977

Determine the values, rounded to the thousandth, of the following probabilities:

(a) $\mathcal{P}(\mathcal{X}\leqslant 60)$ (b) $\mathcal{P}(\mathcal{X}\geqslant 45)$ (c) $\mathcal{P}(48\leqslant\mathcal{X}\leqslant 56)$

E.5540 Consider the random variable $\mathcal{X}$ following a normal distribution with mean 17 and standard deviation type σ *($\mathcal{X}\sim\mathcal{N}(17\,;\sigma^2)$)*.

Knowing that $\mathcal{P}(14\leqslant\mathcal{X}\leqslant 20)=0.382$, determine the value of σ.

We'll use the value table below below, where the random variable $\mathcal{Z}$ follows a centered and reduced normal distribution *($\mathcal{Z}\sim\mathcal{N}(0\,;1)$)*:

x	$-2{,}25$	-1.75	$-0{,}5$	0	0.75	1.25	1.75
$\mathcal{P}(\mathcal{Z}\leqslant x)$	0,012	0.040	0.309	0.5	0.773	0.894	0.960

E.6425 In anticipation of an election, a polling institute collects the voting intentions of future voters. Among the 1200 people who responded to the survey, 52.9 % of voters would vote for the candidate A.

At the confidence threshold of 95 %, can candidate A believe in his victory?

26. *Annales - Normal law and conditional probability*

E.6073 An industrial company manufactures cylindrical parts in large quantities. For any part taken at random, we call $\mathcal{X}$ the random variable that associates its length in millimeters and $\mathcal{Y}$ the random variable that associates its diameter in millimeters.

We assume that $\mathcal{X}$ follows a normal distribution with mean $\mu_1=36$ and standard deviation $\sigma_1=0.2$, and that $\mathcal{Y}$ follows a normal distribution with mean $\mu_2=6$ and standard deviation $\sigma_2=0.05$.

(1) A part is said to be compliant for length if its length is between $\mu_1-3\sigma_1$ and $\mu_1+3\sigma_1$. What is an approximate value, to the nearest 10^{-3}, of the probability p_1 that a randomly selected part will be compliant in terms of length?

(2) A part is said to be compliant in terms of diameter if its diameter is between 5.88 mm and 6.12 mm. The table shown opposite was obtained using a spreadsheet. It shows, for each value of k, the probability that $\mathcal{Y}$ is less than or equal to that value.

k	$\mathcal{P}(\mathcal{Y}\leqslant k)$
5.8	3.16712E-05
5.82	0.000159109
5.84	0.000687138
5.86	0.00255513
5.88	0.008197536
5.9	0.022750132
5.92	0.054799292

k	$\mathcal{P}(\mathcal{Y}\leqslant k)$
5.94	0.11506967
5.96	0.211855399
5.98	0.344578258
6	0.5
6.02	0.655421742
6.04	0.788144601
6.06	0.88493033

k	$\mathcal{P}(\mathcal{Y}\leqslant k)$
6.08	0.945200708
6.1	0.977249868
6.12	0.991802464
6.14	0.99744487
6.16	0.999312862
6.18	0.999840891
6.2	0.999968329

Determine, to the nearest 10^{-3}, the probability p_2 that a randomly selected coin will be compliant for diameter *(. The table opposite can be used to help with this.)*.

(3) A coin is selected at random. We call L the event "*the part is compliant for length*" " and D the event "*the part is compliant for diameter*".

." We assume that events L and D are independent.

(a) A part is accepted if it complies with the length and diameter requirements.
Determine the probability that a randomly selected part will not be accepted *(the result will be rounded to 10^{-2})*.

(b) Justify that the probability of it conforming to the diameter requirement, given that it does not conform to the length requirement, is equal to p_2.

E.5511 The company *Fructidoux* manufactures compotes that it packages in small 50 gram jars. It wants to label them as "*low-calorie compote*".

The law requires that the sugar content, i.e., the proportion of sugar in the compote, be between 0.16 and 0.18. In this case, the small jar of compote is said to be compliant.

The company has two production lines: F_1 and F_2.

Parts A and B can be processed independently.

Part A

The production line F_2 seems more reliable than the production line F_1. However, it is slower.
Thus, in total production, 70 % small pots come from the F_1 line and 30 % from the F_2line.

The F_1 chain produces 5 % non-compliant compotes and the F_2 chain produces 1 %. A small jar is randomly selected from the total production. Consider the events:

- E : "*The small pot comes from the* F_2" line;
- C : "*The small pot is compliant*".

1. Construct a weighted tree on which the above data will be indicated.
2. Calculate the probability of the event: "*the small pot is compliant and comes from production line* F_1".
3. Determine the probability of the event C.
4. Determine, at 10^{-3} close, the probability of event E knowing that event C has occurred.

Part B

1. Note $\mathcal{X}$ la random variable which, to a small jar taken at random from the chain's production F_1, associates its sugar content.

   It is assumed that $\mathcal{X}$ follows the normal distribution with expectation $m_1 = 0.17$ and standard deviation $\sigma_1 = 0.006$.

   In the following, we can use the table below:

α	β	$\mathcal{P}(\alpha \leqslant \mathcal{X} \leqslant \beta)$
0.13	0.15	00004
0.14	0.16	0.0478
0.15	0,17	0.4996
0.16	0.18	0.9044
0.17	0.19	0.4996
0.18	0.20	00478
0.19	0.21	0.0004

   Give an approximate value for 10^{-4} close to the probability that a small pot taken at random from the production of the chain F_1 is compliant.

2. We denote $\mathcal{Y}$ the random variable that associates the sugar content with a small jar taken at random from the production of chain F_2.

   We assume that $\mathcal{Y}$ follows a normal distribution with mean $m_2 = 0.17$ and standard deviation σ_2.

   We also assume that the probability that a small jar taken at random from the production line F_2 is compliant is equal to 0.99.

   Let $\mathcal{Z}$ be the random variable defined by: $Z = \dfrac{\mathcal{Y} - m_2}{\sigma_2}$.

   a. What distribution does the random variable $\mathcal{Z}$ follow?
   b. Determine, based on σ_2 the interval to which $\mathcal{Z}$ belongs when $\mathcal{Y}$ belongs to the interval $[0.16 ; 0.18]$.
   c. Determine the value of σ_2 rounded to 10^{-3} près.

   We can use the table below, in which the random variable $\mathcal{Z}$ follows the normal distribution of expectation 0 and standard deviation 1.

β	$\mathcal{P}(-\beta \leqslant \mathcal{Z} \leqslant \beta)$
2,4324	0.985
2.4573	0.986
2.4838	0.987
2,5121	0.988
2.5427	0.989
2.5758	0.990
2.6121	0.991
2.6521	0.992
2.6968	0.993

27. *Annales - normal distribution and binomial distribution*

E.6075 *All numerical results must be given in decimal form and rounded to three decimal places.*

A factory manufactures spherical balls whose diameter is expressed in millimeters. A ball is considered non-standard when its diameter is less than 9 *mm* or greater than 11 *mm*.

Part A

1. We call $\mathcal{X}$ the random variable that associates each ball chosen at random from production with its diameter expressed in *mm*.

   We assume that the random variable $\mathcal{X}$ follows a normal distribution with mean 10 and standard deviation 0.4.

   Show that the probability that a ball is out of specification, rounded to 0.0001, is 0.0124. The table of values given in the appendix may be used.

2. A production control is put in place such that 98 % non-standard balls are discarded and 99 % correct balls are retained.

   A ball is chosen at random from the production. We note:
   - N the event: "*the selected ball is within the standards*";
   - A the event: "*the selected ball is accepted after the control*".

   a. Construct a weighted tree that combines the data from the statement.
   b. Calculate the probability of the event A
   c. What is the probability that an accepted ball will be non-standard?

Part B

This production control proved too costly for the company, so it was abandoned: from now on, all balls produced are kept and packaged in bags of 100 balls.

The probability that a ball is out of specification is considered to be 0.012 4.

We will assume that taking a random bag of 100 balls is equivalent to drawing 100 balls with replacement from the set of balls produced.

We call $\mathcal{Y}$ the random variable that associates the number of balls that are not standard with each bag of 100 balls.

1. What is the distribution of the random variable $\mathcal{Y}$?
2. What are the expected value and standard deviation of the random variable $\mathcal{Y}$?
3. What is the probability that a bag of 100 balls contains exactly two balls that are not standard?
4. What is the probability that a bag of 100 balls contains at most one ball that is not standard?

d	$\mathcal{P}(\mathcal{X}<d)$
0	$3.06E\text{-}138$
1	$2.08E\text{-}112$
2	$2.75E\text{-}89$
3	$7.16E\text{-}69$
4	$3.67E\text{-}51$
5	$3.73E\text{-}36$
6	$7.62E\text{-}24$
7	$3.19E\text{-}14$

d	$\mathcal{P}(\mathcal{X}<d)$
8	$2.87E-07$
9	0.00620967
10	0.5
11	0.99379034
12	0.99999971
13	1
14	1
15	1

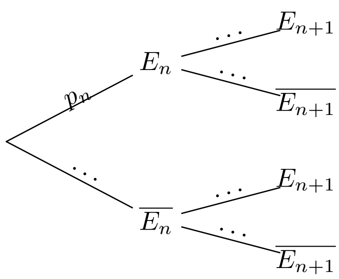
d	$\mathcal{P}(\mathcal{X}<d)$
16	1
17	1
18	1
19	1
20	1
21	1
22	1

E.5431 In a company, we are interested in the probability of an employee being absent during a period of flu epidemic:
- A sick employee is absent.
- The first week of work, the employee is not sick.
- If in week n the employee is not ill, he falls ill in week $n+1$ with probability equal to 0.04.
- If in week n the employee is ill, he remains ill in week $n+1$ with probability equal to 0.24.

For any natural number n greater than or equal to 1, we denote by E_n the event "*the employee is absent due to illness on the n-th semaine*". We note p_n the probability of the event E_n.

We thus have $p_1=0$ and, for any natural number n greater than or equal to 1: $0\leqslant p_n<1$

1. a. Determine the value of p_3 using a probability tree.
   b. Knowing that the employee was absent due to illness in the third week, determine the probability that he was also absent due to illness in the second week.
2. a. Copy onto the copy and complete the probability tree given below:



   b. Show that, for any natural number n greater than or equal to 1: $p_{n+1}=0.2\cdot p_n+0.04$.
   c. Show that the sequence (u_n) defined for any natural number n greater than or equal to 1 by $u_n=p_n-0.05$ is a geometric sequence whose first term and common ratio r will be given. Deduce the expression of u_n, then of p_n in terms of n and r.
   d. Deduce the limit of the sequence (p_n).
   e. In this question, we assume that the sequence (p_n) is increasing. Consider the following algorithm:

Variables:	K and J are natural numbers
	P is a real number.
Initialization:	P takes the value 0.
	J takes the value 1.
Input:	Enter the value of K.
Processing:	While $P<0.05-10^{-K}$
	P takes the value $0.2\times P+0.04$
	J takes the value $J+1$.
	End while
Output:	Display J.

   What does the final display J correspond to??
   Why are we sûr that this algorithm s'arrête?

3. This company employs 220 employees. For the following, it is assumed that the probability of an employee being ill in a given week during this epidemic period is equal to $p=0.05$.

It is assumed that the state of health of an employee does not depend on the state of health of his colleagues.

We denote by $\mathcal{X}$ the random variable that gives the number of sick employees in a given week.

(a) Justify that the random variable $\mathcal{X}$ follows a binomial distribution whose parameters we will give. Calculate the mathematical expectation μ and standard deviation σ of the random variable $\mathcal{X}$.

(b) We admit that the distribution of the random variable $\dfrac{\mathcal{X}-\mu}{\sigma}$ can be approximated by the reduced centered normal distribution, i.e. with parameters 0 and 1.

We denote $\mathcal{Z}$ a random variable following the reduced centered normal distribution. The following table gives the probabilities of the event $Z<x$ for some values of the real number x.

x	-1.55	-1.24	-0.93	-0.62	-0.31	0.00
$\mathcal{P}(\mathcal{Z}<x)$	0.061	0.108	0.177	0.268	0.379	0.500

x	0.31	0.62	0.93	1.24	1.55
$\mathcal{P}(\mathcal{Z}<x)$	0.621	0.732	0.823	0.892	0.939

Calculate, using the approximation proposed in question (b), an approximate value to within 10^{-2} of the probability of the event : "

the number of employees absent from the company during a given week is greater than or equal to 7 and less than or equal to 15".

28. *Annales - Normal laws and fluctuation intervals*

E.6761 On a tennis court, a ball machine allows a player to practice alone. This device sends balls one at a time at a steady pace. The player hits the ball, and then the next ball arrives.

According to the manufacturer's manual, the ball machine sends the ball randomly to the right or left with equal probability.

Throughout the exercise, results will be rounded to the nearest 10^{-3}.

Part A

The player is about to receive a series of 20 balls.

(1) What is the probability that the ball launcher will send 10 balls to the right?

(2) What is the probability that the ball launcher will send between 5 and 10 balls to the right?

Part B

The ball launcher is equipped with a reservoir that can hold 100 balls. In a sequence of 100 throws, 42 balls were thrown to the right.

The player then doubts that the device is working properly. Are his doubts justified?

Part C

To increase the difficulty, the player sets the ball launcher to give the balls a spin. They can either be "*lifted*", or "*cut*".

The probability that the ball tosser will throw a ball to the right is always equal to the probability that the ball tosser will throw a ball to the left

The device settings allow us to state that :

- the probability that the ball launcher will send a topspin ball to the right is 0.24 ;
- the probability that the ball machine will send a slice ball to the left is 0.235.

If the ball machine sends a slice ball, what is the probability that it will be sent to the right?

E.6422 In anticipation of an election between two candidates, A and B, a polling institute is gathering the voting intentions of future voters.

Among the 1200 people who responded to the poll, 47 % say they intend to vote for candidate A and the rest for candidate B.

Given the candidates' profiles, the polling institute estimates that 10 % of the people who say they want to vote for candidate A are not telling the truth and will actually vote for candidate B, while 20 % of those who say they want to vote for candidate B are not telling the truth and will actually vote for candidate A.

A person who responded to the poll is chosen at random and the following is noted :

- A the event "*the respondent says they want to vote for candidate A*" ;
- B the event "*the respondent says they want to vote for candidate B*" ;
- V the event "*the respondent is telling the truth*".

(1) Construct a probability tree reflecting the situation.

(2) (a) Calculate the probability that the respondent is telling the truth.

(b) Assuming that the respondent is telling the truth, calculate the probability that they say they will vote for candidate A

(3) Demonstrate that the probability that the selected person will actually vote for candidate A is 0.529.

(4) The polling institute then publishes the following results :

> 52.9 % of voters* would vote for candidate A.
>
> * estimate after adjustment, based on a survey of a representative sample of 1200 people

At a confidence level of 95 %, can candidate A believe in his victory? *(Approximate values to the nearest thousandth will be used)*

(5) To conduct this poll, the institute carried out a telephone survey at a rate of 10 calls per half hour. The probability that a person contacted will agree to respond to this

survey is 0.4.
The polling institute wants to obtain a sample of 1200 responses.
How much time, expressed in hours, should the institute allow to achieve this goal?

E.6766 In a supermarket, a study is conducted on the sale of fruit juice bottles over a period of one month.

- 40 % of the bottles sold are orange juice bottles ;
- 25 % of the orange juice bottles sold are labeled "*pure juice*".

Among the bottles that are not orange juice, the proportion of bottles labeled "*pure juice*" " is noted as x, where x is a real number in the interval $\left[0\,;1\right]$.
Furthermore, 20 % of the fruit juice bottles sold have the label "*pure juice*".

A bottle of fruit juice is randomly selected from the checkout line. The following events are defined :

- R : "*the bottle selected is a bottle of orange juice*" ;
- J : "*the bottle selected is a bottle of pure juice*".

Part A

1. Represent this situation using a weighted tree.
2. Determine the exact value of x.
3. A bottle scanned at the checkout and selected at random is a bottle of "*pure juice*".
   Calculate the probability that it is a bottle of orange juice.

Part B

In order to gain a better understanding of his customer base, the supermarket manager conducts a study on a batch of 500 recently sold bottles of fruit juice
We note $\mathcal{X}$ the random variable equal to the number of bottles of "*pure juice*" in this batch.

We assume that the stock of bottles in the supermarket is large enough that the selection of these 500 bottles can be considered a random draw with replacement.

1. Determine the distribution of the random variable $\mathcal{X}$. Give the parameters.
2. Determine the probability that at least 75 bottles in this sample of 500 bottles are "*pure juice*". Round the result to three decimal places.

Part C

A supplier claims that 90 % of the bottles of pure orange juice it produces contain less than 2 % pulp. The supermarket's quality control department takes a sample of 900 bottles to verify this claim. Of this sample, 766 bottles contain less than 2 % pulp.

1. Determine the asymptotic fluctuation interval of the proportion of bottles containing less than 2 % pulp at the threshold of 95 %
2. What do you think of the supplier's statement?

E.5538 *The parts A, B and C peuvent can be treated independently of each other autres*

An industrial bakery uses a machine to make farmhouse loaves weighing an average of 400 grams. To be sold to customers, these loaves must weigh at least 385 grammes. A loaf whose mass is strictly less than 385 grammes is a non-commercial loaf, a loaf whose mass is greater than or equal to 385 grammes is marketable.

The mass of a loaf produced by the machine can be modelled by a random variable $\mathcal{X}$ following the normal distribution with expectation $\mu=400$ and standard deviation $\sigma=11$.

The probabilities will be rounded to the nearest thousandth

Part A

The following table, in which values are rounded to the nearest thousandth, may be used

x	308	385	390	395	400	405	410	415	420
$\mathcal{P}(\mathcal{X}\leqslant x)$	0.035	0.086	0.182	0.325	0.5	0.675	0.818	0.914	0.965

1. Calculate $\mathcal{P}\big(390\leqslant\mathcal{X}\leqslant 410\big)$.
2. Calculate the probability p that a bread chosen at random from the production is marketable.
3. The manufacturer finds this probability p too low. He decides to modify his production methods in order to vary the value of σ without changing that of μ.

   For what value of σ is the probability that a loaf of bread is marketable equal to 96 %? Round the result to the nearest tenth.

   We can use the following result : when $\mathcal{Z}$ is a random variable that follows the normal distribution of expectation 0 and standard deviation 1, we have :
   $\mathcal{P}\big(\mathcal{Z}\leqslant -1.751\big)\approx 0.040$

Part B

Production methods have been modified in order to obtain 96 % marketable bread. In order to evaluate the effectiveness of these modifications, quality control is performed on a sample of 300 bread produced.

1. Determine the asymptotic fluctuation interval at the threshold of 95 % of the proportion of marketable loaves in a sample of 300.
2. Of the 300 loaves in the sample, 283 are marketable.
   Given the fluctuation interval obtained in question 1, can we conclude that the objective has been achieved?

Part C

The baker uses an electronic scale. The number of days that this electronic scale operates without malfunctioning is a random variable $\mathcal{T}$ that follows an exponential distribution with parameter λ.

1. We know that the probability that the electronic scale will not malfunction before 30 days is 0.913. Deduce the value of λ rounded to three decimal places.

Throughout the rest of this section, we will take : $\lambda=0.003$

2. What is the probability that the electronic scale will still be working without malfunctioning after 90 days, given that it has been working without malfunctioning for 60 days?
3. The seller of this electronic scale assured the baker that there was a 50

29. Annales - normal laws and confidence intervals

E.6740 ⚠ An institute conducts a survey to determine the proportion of people in a given population who are in favor of a land use planning project. To do this, a random sample of people from this population is surveyed, and each person is asked one question.

The three parts relate to this situation, but can be treated independently.

Part A : Number of people who agree to respond to the survey

In this part, we assume that the probability that a respondent will agree to answer the question is equal to 0.6.

1. The survey institute interviews 700 people. Let $\mathcal{X}$ be the random variable corresponding to the number of respondents who agree to answer the question.
   a. What is the distribution of the random variable $\mathcal{X}$? Justify your answer.
   b. Which of the following numbers is the best approximation of $\mathcal{P}\big(\mathcal{X}\geqslant 400\big)$?
      0.92 ; 0.93 ; 0.94 ; 0.95
2. How many people must the institute survey at a minimum to guarantee, with a probability greater than 0.9, that the number of people responding to the survey will be greater than or equal to 400?

Part B : Proposal for people in favor of the project in the population

In this part, we assume that n people answered the question, and we assume that these people constitute a random sample of size n *(where n is a natural number greater than* 50*)*.
Among these people, 29 % are in favor of the development project.

1. Give a confidence interval, at a confidence level of 95 %, for the proportion of people who are in favor of the project in the total population.
2. Determine the minimum value of the integer n so that the confidence interval, at a confidence level of 95 %, has a range less than or equal to 0.04.

Part C : Correction due to the insincerity of certain responses

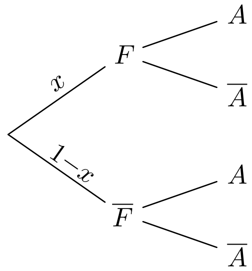
In this section, we assume that, among the respondents who agreed to answer the question, 29 % say they are in favor of the project
The polling institute also knows that the question asked may be embarrassing for respondents, so some of them are not sincere and answer the opposite of their true opinion. Thus, a person who says they are in favor may :

- either actually be in favor of the project if they are sincere.
- or actually be opposed to the project if they are not sincere.

Based on experience, the institute estimates that 15 % of respondents are insincere in their answers and admits that this rate is the same regardless of the respondent's opinion.
The aim of this section is to use this data to determine the actual rate of people in favor of the project, using a probabilistic model. We randomly select the card of a person who has responded, and define :

- F the event "*the person is actually favorable to projet*" ;
- $\overline{F}$ the event "*the person is actually unfavorable to projet*".
- A the event "*the person claims to be favorable to projet*" ;
- $\overline{A}$ the event "*the person claims to be unfavorable to the project*".

Thus, according to the data, we have : $\mathcal{P}\big(A\big)=0.29$.



1. Interpreting the data in the statement, indicate the values $\mathcal{P}_F\big(A\big)$ and $\mathcal{P}_{\overline{F}}\big(A\big)$.
2. We pose : $x=\mathcal{P}\big(F\big)$.
   a. Reproduire on the copy and complete the probability tree opposite.
   b. Deduce an equality verified by x.
3. Determine, among the people who responded to the survey, the proportion who are really in favor of the project.

E.6071

Part A

Let n be a natural number, p a real number between 0 and 1, and $\mathcal{X}_n$ a random variable following a binomial distribution with parameters n and p. Let $\mathcal{F}_n=\dfrac{\mathcal{X}_n}{n}$ and f denote the value taken by f and the value taken by F_n, respectively. Recall that, for n sufficiently large, the interval $\left[p-\dfrac{1}{\sqrt{n}}\,;p+\dfrac{1}{\sqrt{n}}\right]$ contains the frequency f with a probability at least equal to 0.95.

Deduce that the interval $\left[f-\dfrac{1}{\sqrt{n}}\,;f+\dfrac{1}{\sqrt{n}}\right]$ contains p with a probability at least equal to 0.95.

Part B

We want to study the number of students who know the meaning of the acronym URSSAF. To do this, we ask them a multiple-choice questionnaire. Each student must choose from three possible answers.

We denote r as the probability that a student knows the correct answer. Any student who knows the correct answer answers A, otherwise they answer at random *(in an equiprobable manner)*.

1. We survey a student at random. We note:
   - A: "*the student responds A*";
   - B: "*the student answers B*";
   - C: "*the student answers C*";
   - R: "*The student knows the answer*";
   - $\overline{R}$: The opposite event of R

   a. Translate this situation using a probability tree.

   b. Show that the probability of event A is:
   $\mathcal{P}(A)=\dfrac{1}{3}\cdot(1+2\cdot r)$.

   c. Express in terms of r the probability that a person who chose A knows the correct answer.

2. To estimate r, we survey 400 people and note $\mathcal{X}$ the random variable counting the number of correct answers. We will assume that randomly surveying 400 students is equivalent to drawing 400 students with replacement from the set of all students.

   a. Give the distribution of $\mathcal{X}$ and its parameters n and p as a function of r.

   b. In an initial survey, we find that 240 students respond A, out of the 400 surveyed.
   Give a confidence interval at the threshold of 95 % for the estimate of p.
   Deduce a confidence interval at the 95 % threshold of r

   c. In the following, it will be assumed that r=0.4. Given the large number of students, $\mathcal{X}$ will be assumed to follow a normal distribution.
   - Give the parameters of this normal distribution.
   - Donner an approximate value of $\mathcal{P}(\mathcal{X}\leqslant 250)$ to the nearest 10^{-2}.

     We can use the table below 1, which gives an approximate value of $\mathcal{P}(\mathcal{X}\leqslant t)$ where $\mathcal{X}$ is the random variable from question 2 b.

$C3$ | =LOI.NORMALE($A12+E$1 ;240 ;RACINE(96) ;VRAI)

	A	B	C	D	E	F	G	H	I	J	K
1	0,0	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
2	235	0,305	0,309	0,312	0,316	0,319	0,323	0,327	0,33	0,334	0,338
3	236	0,342	0,345	0,349	0,353	0,357	0,36	0,364	0,368	0,372	0,376
4	237	0,38	0,384	0,388	0,391	0,395	0,399	0,403	0,407	0,411	0,415
5	238	0,419	0,423	0,427	0,431	0,435	0,439	0,443	0,447	0,451	0,455
6	239	0,459	0,463	0,467	0,472	0,476	0,48	0,484	0,488	0,492	0,496
7	240	0,5	0,504	0,508	0,512	0,516	0,52	0,524	0,528	0,533	0,537
8	241	0,541	0,545	0,549	0,553	0,557	0,561	0,565	0,569	0,573	0,577
9	242	0,581	0,585	0,589	0,593	0,597	0,601	0,605	0,609	0,612	0,616
10	243	0,62	0,624	0,628	0,632	0,636	0,64	0,643	0,647	0,651	0,655
11	244	0,658	0,662	0,666	0,67	0,673	0,677	0,681	0,684	0,688	0,691
12	245	0,695	0,699	0,702	**0,706**	0,709	0,713	0,716	0,72	0,723	0,726
13	246	0,73	0,733	0,737	0,74	0,743	0,746	0,75	0,753	0,756	0,759
14	247	0,763	0,766	0,769	0,772	0,775	0,778	0,781	0,784	0,787	0,79
15	248	0,793	0,796	0,799	0,802	0,804	0,807	0,81	0,813	0,815	0,818
16	249	0,821	0,823	0,826	0,829	0,831	0,834	0,836	0,839	0,841	0,844
17	250	0,846	0,849	0,851	0,853	0,856	0,858	0,86	0,863	0,865	0,867
18	251	0,869	0,871	0,874	0,876	0,878	0,88	0,882	0,884	0,886	0,888
19	252	0,89	0,892	0,893	0,895	0,897	0,899	0,901	0,903	0,904	0,906
20	253	0,908	0,909	0,911	0,913	0,914	0,916	0,917	0,919	0,921	0,922
21	254	0,923	0,925	0,926	0,928	0,929	0,931	0,932	0,933	0,935	0,936
22	255	0,937	0,938	0,94	0,941	0,942	0,943	0,944	0,945	0,947	0,948
23	256	0,949	0,95	0,951	0,952	0,953	0,954	0,955	0,956	0,957	0,958
24	257	0,959	0,96	0,96	0,961	0,962	0,963	0,964	0,965	0,965	0,966
25	258	0,967	0,968	0,968	0,969	0,97	0,97	0,971	0,972	0,972	0,973
26	259	0,974	0,974	0,975	0,976	0,976	0,977	0,977	0,978	0,978	0,979
27	260	0,979	0,98	0,98	0,981	0,981	0,982	0,982	0,983	0,983	0,984

Extrait of a sheet of calcul

Exemple use: at the intersection of row 12 and column E the number 0.706 corresponds to $\mathcal{P}(\mathcal{X}\leqslant 245.3)$.

E.6767 **Part A**

Chikungunya is a viral disease transmitted from one human to another through the bites of infected female mosquitoes.

A test has been developed to screen for this virus. The laboratory that manufactures this test provides the following characteristics:

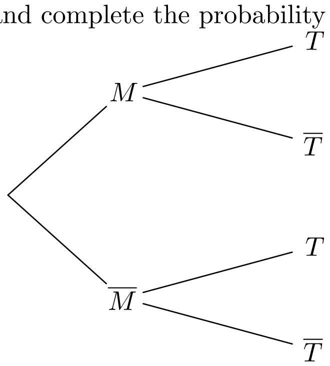
- the probability that a person infected with the virus will test positive is 0.98;
- the probability that a person not infected with the virus will test positive is 0.01.

A systematic screening test is carried out in a "*target*". population. An individual is chosen at random from this population. We call:

- M the event: "*the individual chosen is infected with chikungunya*"
- T the event: "*the test of the individual chosen is positive*"

We note $\overline{M}$ *(respectively* $\overline{T}$*)* the opposite event of the event M *(respectively* T*)*.
Note p $(0\leqslant p\leqslant 1)$ the proportion of people affected by the disease in the target population.

1 a Copy and complete the probability tree below:



b Express $\mathcal{P}\big(M\cap T\big)$, $\mathcal{P}\big(\overline{M}\cap T\big)$ and then $\mathcal{P}\big(T\big)$ as a function of p.

2 a Demonstrate that the probability of M given T is given by the function f defined on $\big[0\,;1\big]$ by:

$$f(p)=\frac{98\cdot p}{97\cdot p+1}$$

b Study the variations of the function f

3 The test is considered reliable when the probability that a person with a positive test result actually has chikungunya is greater than 0.95.
Using the results of question 2, , at what proportion p of patients in the population is the test reliable?

Part B

In July 2014, the health surveillance institute of an island, based on data reported by doctors, published that 15 % of the population was infected with the virus.

As some people do not necessarily consult their doctor, it is thought that the proportion is actually higher.
To verify this, it is proposed to study a sample of 1 000 people chosen at random from the island. The population is large enough to consider that such a sample is the result of sampling with replacement.

We denote by $\mathcal{X}$ the random variable which, to any sample of 1000 people chosen at random, maps the number of people affected by the virus and by $\mathcal{F}$ the random variable giving the frequency associée

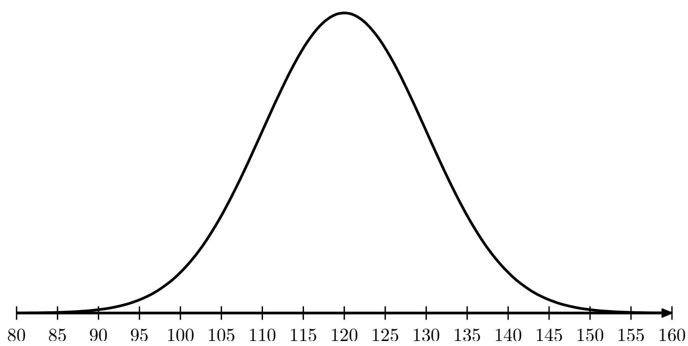
1 a Sous the hypothesis $p=0.15$, determine the law of $\mathcal{X}$.

b In a sample of 1000 people chosen at random on the island, there are 197 people affected by the virus.
What conclusion can be drawn from this observation about the figure of 15 % published by the Institut de veille sanitaire?
Justify. *(We can use the calculation of a fluctuation interval at the threshold of* 95 %*)*

2 We now consider that the value of p is unknown.
Using the sample from question 1 b, propose a confidence interval for the value of p, at the confidence level of 95 %.

Part C

The incubation time, expressed in hours, of the virus can be modeled by a random variable $\mathcal{T}$ following a normal distribution of standard deviation $\sigma=10$.
We wish to determine its mean μ.
The graphical representation of the probability density function $\mathcal{T}$ is given below:



1 a Using the graph, estimate an approximate value for μ

b We are given $\mathcal{P}\big(\mathcal{T}\leqslant 110\big)=0.18$. Shade the area of the graph corresponding to the given probability.

2 Let $\mathcal{T}'$ be the random variable equal to $\dfrac{\mathcal{T}-\mu}{10}$.

a What distribution does the random variable $\mathcal{T}'$ follow?

b Determine the mean μ of the random variable $\mathcal{T}$ rounded to the nearest whole number and check the conjecture in question 1.

30. *Unclassified financial years*

E.6957 An institute conducts a survey to find out the proportion of people in a given population who are in favor of a land development project. To do this, a random sample of the population is interviewed, and each person is asked a question.

It is assumed that the probability of a respondent agreeing to answer the question is 0.6.

How many people must the institute interview as a minimum to guarantee, with a probability greater than 0.9, that the number of people responding to the survey is greater than or equal to 400.

E.7579 Consider the random variable $\mathcal{X}$ following a normal distribution with parameter 13 and 1 $(\mathcal{X} \sim \mathcal{N}(13\,;1^2))$.

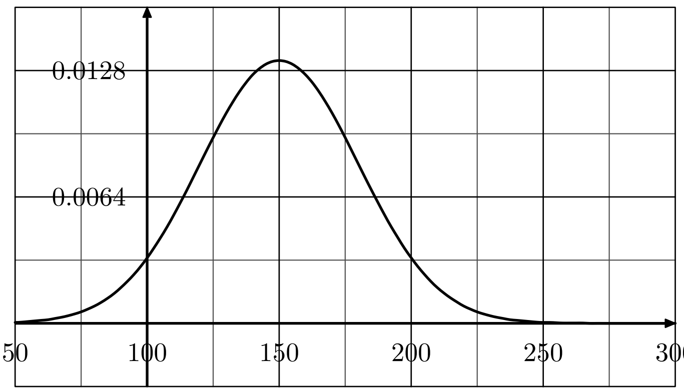
The probability value $\mathcal{P}(11.5 \leqslant \mathcal{X} \leqslant 14.5)$ rounded to the nearest thousandth:

1. 0.437
2. 0.866
3. 0.954
4. 0.999

E.5564 We are interested in a species of fish present in an area of the planet.

We denote $\mathcal{X}$ the random variable which, to each fish observed in this area, associates its size in cm.

A statistical study has shown that the random variable $\mathcal{X}$ follows a normal distribution with mean μ and standard deviation $\sigma = 30$. The probability density curve associated with $\mathcal{X}$ is shown below:



1. By graphical reading, give the value of μ.
2. One of these fish is caught at random. Give the probability, rounded to 10^{-2}, of having a fish whose size is between 150 cm and 210 cm.
3. A fish of this species is considered adult when it measures more than 120 cm.

   A fish of the species under consideration is caught. Give the probability, rounded to 10^{-2}, of catching an adult fish.

E.5509 Let $\mathcal{X}$ be a random variable following a probability law $\mathcal{N}(0\,;1)$. For any real number $\alpha \in]0\,;1[$, there exists a single positive real u_α such that:

$$\mathcal{P}(-u_\alpha \leqslant \mathcal{X} \leqslant u_\alpha) = 1 - \alpha$$

E.5510 Let α be a real number belonging to $]0\,;1[$ and $\mathcal{Z}$ a random variable following the normal centered reduced distribution $(\mathcal{Z} \sim \mathcal{N}(0\,;1))$.

We note u_α the unique positive real verifying the equality:

$$\mathcal{P}(-u_\alpha \leqslant \mathcal{Z} \leqslant u_\alpha) = 1 - \alpha$$

Let $\mathcal{X}_n$ be a variable following a binomial distribution $\mathcal{B}(n\,;p)$.

Consider the random variable $F_n = \dfrac{\mathcal{X}_n}{n}$ associated with the frequency of success.

Set the following limit:

$$\lim_{n \mapsto +\infty} \mathcal{P}\left(p - u_\alpha \frac{\sqrt{p \cdot (1-p)}}{\sqrt{n}} \leqslant F_n \leqslant p + u_\alpha \frac{\sqrt{p \cdot (1-p)}}{\sqrt{n}}\right) = 1 - \alpha$$

E.6070 Let n be a natural number, p a real number between 0 and 1, and $\mathcal{X}_n$ a random variable following a binomial distribution with parameters n and p. Let $\mathcal{F}_n = \dfrac{\mathcal{X}_n}{n}$ and f denote the value taken by f and the value taken by F_n, respectively. Recall that, for n sufficiently large, the interval $\left[p - \dfrac{1}{\sqrt{n}}\,;p + \dfrac{1}{\sqrt{n}}\right]$ contains the frequency f with a probability at least equal to 0.95.

Deduce that the interval $\left[f - \dfrac{1}{\sqrt{n}}\,;f + \dfrac{1}{\sqrt{n}}\right]$ contains p with a probability at least equal to 0.95.

E.5568 A student must go to school every morning for $8h\,00min.$. To do so, he uses a bicycle.

We model his travel time, expressed in minutes, between his home and his high school using a random variable $\mathcal{T}$ that follows a normal distribution with mean $\mu = 17$ and standard deviation $\sigma = 1.6$

To answer the questions in the exercise, we will use the table of values below, where the random variable $\mathcal{Z}$ follows a reduced centered normal distribution:

x	-2	-1.875	-1.75	-1.625	-1.5	-1.375	-1.25
$\mathcal{P}(\mathcal{Z} \leqslant x)$	0.023	0.030	0,040	0.052	0.067	0.084	0.106

x	1.25	1.375	1.5	1.625	1,75	1.875	2
$\mathcal{P}(\mathcal{Z} \leqslant x)$	0.089	0,915	0.933	0.948	0.960	0.970	0.977

Probabilities will be rounded to the nearest thousandth.

1. He leaves home on his bike at $7h\,40min$. What is the probability that he will be late for school?
2. Determine the probability that the student will take between 15 and 20 minutes to get to school.

E.6428 An oyster farmer claims that 60 % of his oysters have a mass greater than 91 g. A restaurateur would like to buy a large quantity of oysters from him, but would first like to verify the oyster farmer's assertion.

The restaurateur buys 10 dozen oysters from this oyster farmer, which we will consider as a sample of 120 oysters drawn at random. His production is large enough to be treated as a random draw.
He finds that 65 of these oysters have a mass greater than 91 g.

Let $\mathcal{F}$ be the random variable that to any sample of 120 oysters associates the frequency of those with a mass greater than 91 g.

After checking the conditions of application, give an asymptotic fluctuation interval at the threshold of 95 % of the random variable $\mathcal{F}$. *(values approximated to the nearest thousandth will be used).*

E.6094 A major cosmetics brand is launching a new moisturizing cream.

The brand would like to sell the new cream in packaging from 50 $m\ell$.
A jar of cream is said to be non-compliant if it contains less than 49 $m\ell$ of cream.

Several series of tests lead to modeling the quantity of cream, expressed in $m\ell$, contained in each jar by a random variable $\mathcal{X}$ following a normal distribution with expectation $\mu=50$ and standard deviation σ.

We note $\mathcal{Z}$ the variable equal to $\dfrac{\mathcal{X}-50}{\sigma}$.

1. Specify the law followed by the random variable $\mathcal{Z}$.
2. Determine a value, rounded to the thousandth, of the real u such that: $\mathcal{P}\big(Z\leqslant u\big)=0.06$
3. We know that the probability of a randomly selected jar being non-compliant is 0.06.

   Deduce the value, rounded to the thousandth, of the standard deviation σ.

