1. Algebraic manipulations (for heredity)

E.5810 Establish the following equality for any natural number n different from 6:

$$\frac{\left(\frac{1}{3}\cdot n+4\right)-6}{n-6}=\frac{1}{3}$$

E.5805 Establish the identity below for any natural number n:

$$\frac{1}{2-\frac{n}{n+1}}=\frac{n+1}{(n+1)+1}$$

E.5811 Establish the following identity for any natural number n:

$$\frac{\frac{10\cdot n+1}{n+10}-1}{\frac{10\cdot n+1}{n+10}+1}=\frac{9}{11}\times\frac{n-1}{n+1}$$

E.5809 Establish the following identity for any natural number n:

$$\frac{n\cdot\frac{1+(0.5)^n}{n}+1}{2\cdot(n+1)}=\frac{1+(0.5)^{n+1}}{n+1}$$

E.5807 Establish the following identity for any natural number n:

$$\frac{(n+2)\cdot(1+2\cdot n)+1}{n+1}=1+2\cdot(n+1)$$

E.5808 Establish the following identity for any natural number n:

$$\frac{n\cdot(n+1)\cdot(2n+1)}{6}+(n+1)^2=\frac{(n+1)(n+2)(2n+3)}{6}$$

E.5806 Establish the following identity for any natural number n:

$$\left(1+\frac{2}{n+1}\right)\cdot(4\cdot n^2+12\cdot n+5)+\frac{6}{n+1}$$
$$=4\cdot(n+1)^2+12\cdot(n+1)+5$$

E.6168 Establish the following equality for any natural number n:

$$\frac{3\times\frac{3^n}{3^n+1}}{1+2\times\frac{3^n}{3^n+1}}=\frac{3^{n+1}}{3^{n+1}+1}$$

2. Introduction to reasoning by recurrence

E.3437

1. Let (u_n) be a sequence whose rank term n is defined, for any natural number n, by:
$$u_n=\frac{2\cdot n}{n+1}$$
Give the simplified expression of the terms u_{n+1} and u_{n+2} as a function of n.

2. Let (v_n) be a sequence whose rank term n can be written as a function of n:
$$v_n=3^{n-1}+4^{n+1}\quad\text{for any natural number } n.$$
Give an expression for v_{n+1} and v_{n+2} as a function of n.

3. For any non-zero natural number n, we have the equality:
$$1^2+2^2+\cdots+n^2=\frac{n\cdot(n+1)\cdot(2n+1)}{6}$$
Give the writing of this identity at rank $(n+1)$.

E.6129 Consider the sequence defined by:

$$u_0=7\quad;\quad u_{n+1}=\frac{1}{4}\cdot u_n+\frac{3}{2}$$

1. Calculate the values of the first six terms of the sequence (u_n), then their values rounded to the nearest thousandth.

2. Note that these first terms verify the property: $u_n\geqslant 2$ for all $n\in\mathbb{N}$.

   How can we justify that all the terms of the sequence (u_n) verify this property?

E.5176 Consider the sequence $(u_n)_{n\in\mathbb{N}}$ defined by the recurrence relation and verifying the conditions:

$$u_0=1\quad;\quad u_1=4\quad;\quad u_{n+2}=2\cdot u_{n+1}-u_n\quad\text{pour all } n\in\mathbb{N}$$

1. (a) Déterminer the first five terms of the sequence (u_n).
   (b) Make a conjecture as to the nature of the sequence (u_n).
   (c) What remains to be shown to establish this recurrence?

2. (a) Give the simplified expression of $u_{n+2}-u_{n+1}$.
   (b) Is this enough to justify the conjecture?

E.3430 ⚠ Let the sequence (u_n) be defined for any natural number n by:

$$u_0=\frac{1}{2}\quad;\quad u_{n+1}=\frac{1}{2}\cdot\left(u_n+\frac{2}{u_n}\right)\quad\text{for any } n\in\mathbb{N}$$

1. Consider the sequence f defined on $]0\,;+\infty[$ by:
$$f(x)=\frac{1}{2}\cdot\left(x+\frac{2}{x}\right)$$
Draw up the table of variation of the function f.

2. Can we conjecture a minoration of the sequence (u_n) for terms of rank greater than or equal to 2.

3. Recurrence - inequalities

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E.5802 Consider the numerical sequence (u_n) defined on $\mathbb{N}$ by:

$$u_0 = 1 \quad ; \quad u_{n+1} = \frac{9}{6-u_n} \quad \text{for all } n\in\mathbb{N}$$

Demonstrate by recurrence that, for any natural number n, we have the framing: $0<u_n<3$.

E.3428 Consider the sequence $(u_n)_{n\in\mathbb{N}}$ defined by:

$$u_0 = 3 \quad ; \quad u_{n+1} = \frac{3}{2}\cdot u_n - 1 \quad \text{for all } n\in\mathbb{N}$$

1. Establish that, for any natural number n, we have: $u_n \geqslant 3$
2. Deduce that the sequence (u_n) is increasing.

E.5033 Consider the sequence (u_n) defined on $\mathbb{N}$ by:

$$u_0 = -3 \quad ; \quad u_{n+1} = \sqrt{3+\frac{u_n}{2}} \quad \text{for all } n\in\mathbb{N}.$$

1. Determine the value of the term of rank 1 of the sequence (u_n).
2. Show that, for any $n\in\mathbb{N}^*$, we have the frame: $0 \leqslant u_n \leqslant 2$

E.3286 The sequence (u_n) is defined by:

$$u_0 = 1 \quad ; \quad u_{n+1} = \frac{1}{2}u_n + n - 1, \text{ for all } n\in\mathbb{N}.$$

Demonstrate that for any $n\geqslant 3$, we have: $u_n \geqslant 0$.

E.4102

1. Show that for any natural number greater than or equal to 3, we have: $2\cdot n^2 \geqslant (n+1)^2$
2. Show by recurrence that for any integer n greater than or equal to 4: $2^n \geqslant n^2$

4. Recurrence - equalities

E.6130 Consider the sequence (u_n) defined on $\mathbb{N}$ by:

$$u_0 = 0 \quad ; \quad u_{n+1} = u_n + 2n + 2 \quad \text{for all } n\in\mathbb{N}$$

Show, using recursive reasoning, that the term of rank n of the sequence (u_n) can be expressed as: $u_n = n^2 + n$

E.3295 Consider the sequence (u_n) of natural numbers defined by:

$$\begin{cases} u_0 = 14 \\ u_{n+1} = 5u_n - 6 \quad \text{for any } n\in\mathbb{N} \end{cases}$$

Show by recurrence that, for any natural number n: $2u_n = 5^{n+2} + 3$.

E.6131 Consider the sequence (u_n) defined for any natural number n by:

$$u_0 = 2 \quad ; \quad u_{n+1} = \frac{1}{5}\cdot u_n + 3\times 0.5^n.$$

Establish, using reasoning by recurrence, the following equality for any natural number n:

$$u_n = -8\times\left(\frac{1}{5}\right)^n + 10\times 0.5^n$$

E.6827 Let (u_n) be the sequence defined by its first term $u_0=5$ and, for any natural number n by:

$$u_{n+1} = 0.5\cdot u_n + 0.5\cdot n - 1.5$$

Using reasoning by recurrence, show that: $u_n = 10\times 0.5^n + n - 5$

E.6950 Consider the sequence (u_n) defined by:

$$u_0 = 3 \quad ; \quad u_{n+1} = 9\times 2^n - u_n \quad \text{for all } n\in\mathbb{N}$$

Show, using recursive reasoning, that the sequence (u_n) is a geometric sequence, specifying its first term and common ratio.

E.3292 Consider the sequence u defined by:

$$\begin{cases} u_0 = 0 \\ u_{n+1} = \dfrac{1}{2-u_n} \end{cases} \quad \text{for all } n\in\mathbb{N}.$$

Demonstrate, using reasoning by recurrence, that for any natural number n, we have: $un = \dfrac{n}{n+1}$.

E.5801 Consider the sequence (u_n) defined by:

$$u_1 = \frac{3}{2} \quad ; \quad u_{n+1} = \frac{n\cdot u_n + 1}{2(n+1)} \quad \text{for all integers } n\in\mathbb{N}^*$$

Show, using recursive reasoning, that for all non-zero natural integers n, we have: $u_n = \dfrac{1+(0.5)^n}{n}$

E.3438 For $x\neq 1$, show that for all $n\in\mathbb{N}$, we have:

$$1 + x + x^2 + \cdots + x^n = \frac{1-x^{n+1}}{1-x}$$

E.3425 Establish the following property, using reasoning by recurrence for any $n\geqslant 1$:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

E.6152 Consider the sequence (w_n) whose terms verify, for any natural number $n\geqslant 1$:

$$w_0 = 1 \quad ; \quad n\cdot w_n = (n+1)\cdot w_{n-1} + 1 \quad \text{for any } n\in\mathbb{N}^*$$

Demonstrate by recurrence that we have the following relationship for any non-zero natural number n: $w_{n+1} - w_n = 2$

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E.4307 Consider the sequence (u_n) defined, for any $n\in\mathbb{N}$, by:

$$u_0=0 \quad ; \quad u_1=1 \quad ; \quad u_{n2}=\frac{u_{n+1}+u_n}{2} \quad \text{for } n\geqslant 0.$$

Demonstrate by using reasoning by recurrence that, for any natural number n:

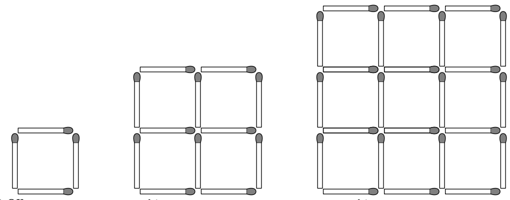
$$u_{n+1}=-\frac{1}{2}\cdot u_n+1$$

E.4645 Consider the sequence (u_n) defined on $\mathbb{N}$ by the following recurrence relation:

$$u_0=0 \quad ; \quad u_{n+1}=u_n-2n+11 \quad \text{for all } n\in\mathbb{N}$$

1. Using the software of your choice, plot the scatterplot associated with the first 15 terms of this sequence.
2. Make a conjecture as to the nature of the curve passing through these points.
3. Using three selected points on this curve, determine the expression of the function f realizing the equality below for the three abscissas of these points:
   $u_n=f(n)$
4. Establish, by recurrence, the expression of the terms of the sequence (u_n) as a function of their rank n.

E.6133 Consider the following constructions:



1er étape 2ième étape 3ième étape

Note (u_n) the numerical sequence defined on $\mathbb{N}^*$ where u_n represents the number of matches required to construct the $n^{\text{ième}}$ step.

1. Determine a recurrence relationship between a term in the sequence (u_n) and its predecessor.
2. Demonstrate, using reasoning by recurrence, that the term of rank n of the sequence (u_n) admits as expression:
   $u_n=2\cdot n^2+2\cdot n$

5. *Recurrence - problems*

E.3290 Consider the function f defined by:

$$f(x)=\frac{2x+1}{x+1}$$

We admit the following properties of the function f:

- The function f is increasing on $[1\,;2]$
- If $x\in[1\,;2]$, then we have $f(x)\in[1\,;2]$

We define the sequence (v_n) defined by:

$$v_0=2 \quad ; \quad v_{n+1}=f(v_n) \quad \text{for all } n\in\mathbb{N}.$$

Establish, by reasoning by recurrence, the following two properties of the suite (v_n):

1. For any natural number n: $1\leqslant v_n\leqslant 2$.
2. For any natural number n: $v_{n+1}\leqslant v_n$.

E.5733 Consider the sequence (u_n) defined by:

$$u_0=2 \quad ; \quad u_{n+1}=\frac{1+3u_n}{3+u_n} \quad \text{for any natural number } n.$$

All terms in this sequence are assumed to be definite and strictly positive.

1. Demonstrate by recurrence that, for any natural number, we have: $u_n>1$.
2. a. Establish that, for any natural number n, we have:
   $$u_{n+1}-u_n=\frac{(1-u_n)(1+u_n)}{3+u_n}$$
   b. Determine the direction of variation of the sequence (u_n).

E.3279 We place ourselves in an orthonormal reference frame and, for any natural number n, we define the points (A_n) by their coordinates $(x_n\,;y_n)$ as follows:

$$\begin{cases} x_0 = -3 \\ y_0 = 4 \end{cases} \quad ; \quad \begin{cases} x_{n+1} = 0.8\cdot x_n - 0{,}6\cdot y_n \\ y_{n+1} = 0.6\cdot x_n + 0.8\cdot y_n \end{cases}$$

For any natural number n, show that the point A_n belongs to the circle with center O and radius 5.

E.5736 Let the numerical sequence (u_n) be defined on $\mathbb{N}$ by:

$$u_0=2 \quad ; \quad u_{n+1}=\frac{2}{3}\cdot u_n+\frac{1}{3}\cdot n+1 \text{ for all } n\in\mathbb{N}$$

1. a. Calculate u_1, u_2, u_3 and u_4, then give their values rounded to the nearest 10^{-2}.
   b. Formulate a conjecture about the direction of variation of this sequence.
2. a. Show that for any natural number n:
   $u_n\leqslant n+3$
   b. Show that for any natural number n:
   $$u_{n+1}-u_n=\frac{1}{3}\cdot(n+3-u_n)$$
   c. Deduce a validation of the previous conjecture.

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E.3423 Consider the sequence (w_n) whose terms satisfy, for any integer $n\geqslant 1$:

$$w_0 = 1 \quad ; \quad n\cdot w_n = (n+1)\cdot w_{n-1} + 1 \text{ for all } n\in\mathbb{N}^*.$$

1. Complete the table of values for the sequence (w_n) below:

w_0	w_1	w_2	w_3	w_4	w_5	w_6
1						

2. a. Make a conjecture about the nature of the sequence (w_n) and its characteristics.
   b. Write the recurrence relation giving $n\cdot w_n$ for rank $(n+1)$.
   c. Establish, using recursive reasoning, that the sequence (w_n) is arithmetic; specify the characteristic elements of this sequence.

E.3419 Consider the sequence $(u_n)_{n\in\mathbb{N}}$ defined by:

$$\begin{cases} u_0 = 5 \\ u_n = \left(1+\dfrac{2}{n}\right)\cdot u_{n-1} + \dfrac{6}{n} \quad \text{for any integer } n\geqslant 1 \end{cases}$$

1. a. Calculate u_1.
   b. The values of u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11} are respectively equal to:
   45, 77, 117, 165, 221, 285, 357, 437, 525, 621.
   From this data conjecture the nature of the sequence $(d_n)_{n\in\mathbb{N}}$ defined by: $d_n = u_{n+1} - u_n$.
2. Consider the arithmetic sequence $(v_n)_{n\in\mathbb{N}}$ of reason 8 and first term $v_0=16$.
   Justify that the sum of the n first terms of this sequence is equal to $4n^2+12n$.
3. Demonstrate by recurrence that for any natural number n, we have:
   $u_n = 4n^2 + 12n + 5$
4. Validate the conjecture made in question 1 b.

E.6041 Consider the sequence (u_n) defined for any non-zero natural number n defined by:

$$u_1 = 7 \quad ; \quad u_{n+1} = \frac{n+2}{n}\cdot u_n - 3 \quad \text{for all } n\in\mathbb{N}^*$$

1. Let (v_n) be the sequence defined by:
   $v_n = \dfrac{u_n}{n}$ for $n\in\mathbb{N}^*$.
   Show that the sequence (v_n) is an arithmetic sequence whose first term and reason are to be specified.
2. Deduce an expression for the terms of the sequence (u_n) as a function of n.

6. *Strong recurrence*

E.6203 Consider the sequence defined by the relations:

$u_0=3 \quad ; \quad u_1=8 \quad ; \quad u_{n+1}=5\cdot u_n-6\cdot u_{n-1}$ for any $n\in\mathbb{N}^*$

1. Determine the values of the terms u_2 and u_3.
2. Using reasoning by recurrence, show that for any non-zero natural number n, we have:
   $u_n = 2^n + 2\times 3^n$

E.6867 Consider the sequence (u_n) defined for any natural number n by:

$$u_0 = 0 \quad ; \quad u_1 = 2 \quad ; \quad u_{n+1} = 4\cdot u_n - 4\cdot u_{n-1} \quad \forall n\in\mathbb{N}^*$$

Show, using reasoning by recurrence, that the terms of the sequence (u_n) admit as expression:

$u_n = n\cdot 2^n$

E.3439 Using reasoning by recurrence, show the following equality:

$$\sum_{k=1}^{n}(-1)^k\cdot k^2 = (-1)^n\cdot\sum_{k=1}^{n} k$$

7. *Limits and recurrences*

E.5819 **Part A**

Consider the function f defined on $\mathbb{R}_+$ by:

$$f(x) = \frac{x+2}{2\cdot x+1}$$

1. Determine the limit of the function f in $+\infty$.
2. Draw up the table of variations of the function f.
3. Justify that the function f is strictly positive on $\mathbb{R}_+$

Part B

Consider the sequence (u_n) defined on $\mathbb{N}$ by:

$$u_0 = 2 \quad ; \quad u_{n+1} = \frac{u_n+2}{2\cdot u_n+1} \text{ for any integer } n\in\mathbb{N}$$

1. Using reasoning by recurrence, establish that for any natural number n, we have the frame:
   $\dfrac{1}{2} \leqslant u_n \leqslant 2$
2. a. Establish that for any natural number n:
   $u_{n+1} - 1 = \dfrac{-u_n+1}{2\cdot u_n+1}$
   b. Demonstrate by recurrence that for any natural number n, $u_n - 1$ has the same sign as $(-1)^n$.

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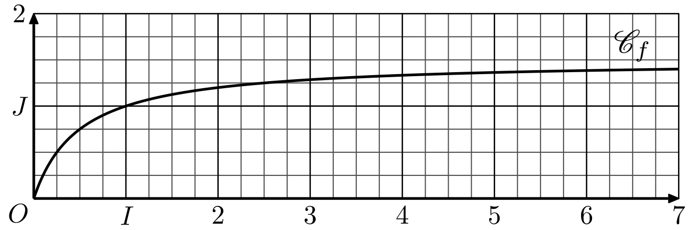
E.5748 Consider the function f defined on $\mathbb{R}_+$ by the following relationship:

$$f(x) = \frac{3{\cdot}x}{1+2{\cdot}x}$$

Note $\mathscr{C}_f$ the representative curve of the function f in a reference frame $(O\,;I\,;J)$ whose representation is given below:



1. a. Justify that the function f is increasing on $\mathbb{R}^+$.
   b. Justify that the curve $\mathscr{C}_f$ admits an asymptote at $+\infty$.
   c. Determine the reduced equation of the tangent (T) to the curve $\mathscr{C}_f$ at the point of abscissa 1.
   d. Consider the line (d), first bisector of the plane, of equation $y=x$. Study the position of the curve $\mathscr{C}_f$ relative to the line (d).
2. Consider the sequence (u_n) defined by the relations:
   $$u_0 = \frac{1}{2} \quad ; \quad u_{n+1} = \frac{3{\cdot}u_n}{1+2{\cdot}u_n}$$
   a. Demonstrate, using reasoning by recurrence, that we have the following framing:
   $$0 \leqslant u_n \leqslant u_{n+1} \leqslant 1 \quad \text{for any natural number } n.$$
   b. Demonstrate, using reasoning by recurrence, the following equality:
   $$u_n = \frac{3^n}{3^n+1} \quad \text{for any natural number } n.$$
   c. Deduce the limit of the sequence (u_n).

E.3583 We define the real sequences $(u_n)_{n\geqslant 0}$ and $(v_n)_{n\geqslant 0}$ by:

$$u_0 = 2 \quad ; \quad \text{for all } n\in\mathbb{N} \quad \begin{cases} u_{n+1} = \dfrac{{u_n}^2+5}{2{\cdot}u_n} \\ v_n = \dfrac{u_n-\sqrt{5}}{u_n+\sqrt{5}} \end{cases}$$

We assume that the sequences (u_n) and (v_n) are defined on $\mathbb{N}$. That is, for every natural number n, we have:

$$u_n \neq 0 \quad ; \quad v_n \neq -\sqrt{5}$$

1. Show that for all $n\geqslant 0$, we have $v_{n+1} = {v_n}^2$. Deduce the relation: $v_n = {v_0}^{(2^n)}$ for all $n\geqslant 0$.
2. Show that $v_0 = \dfrac{-1}{(2+\sqrt{5})^2}$ and deduce the upper bound:
   $$|v_0| \leqslant \frac{1}{16}.$$
   Then determine the limit of the sequence $(v_n)_{n\geqslant 0}$, then that of the sequence $(u_n)_{n\geqslant 0}$ when n tends towards $+\infty$.

E.3420 We define:
- the sequence (u_n) by:
$$\begin{cases} u_0 = 13 \\ u_{n+1} = \dfrac{1}{5}{\cdot}u_n + \dfrac{4}{5} \quad \text{for any natural number } n \end{cases}$$
- the sequence (S_n), for any natural number n, by:
$$S_n = \sum_{k=0}^{n} u_k = u_0 + u_1 + \cdots + u_n$$

1. Show by recurrence that, for any natural number n:
   $$u_n = 1 + \frac{12}{5^n}.$$
   Deduce the limit of the sequence (u_n).
2. a. Determine the direction of variation of the sequence (S_n).
   b. Calculate S_n as a function of n.
   c. Determine the limit of the sequence (S_n).

E.5831 Consider the sequence $(u_n)_{n\in\mathbb{N}}$ defined by:

$$u_0 = 1 \quad ; \quad u_{n+1} = \frac{1}{3}{\cdot}u_n + n - 2 \quad \text{for everything } n\in\mathbb{N}$$

1. Using reasoning by recurrence, establish that, for any natural number n, we have:
   $$u_n = \frac{25}{4}{\cdot}\Big(\frac{1}{3}\Big)^n + \frac{3}{2}{\cdot}n - \frac{21}{4}$$
2. Let S_n be the sum defined for any natural number n by $S_n = \displaystyle\sum_{k=0}^{n} u_k$. Determine the expression of S_n as a function of n.
3. Consider the sequence (L_n) defined by:
   $$L_n = \frac{S_n}{n} \quad \text{for any natural number } n.$$
   Determine the value of the limit of the sequence (L_n).

E.3708

1. Let the numerical sequences (u_n) and (v_n) be defined for any $n\in\mathbb{N}$ by:
   $$u_0 = 1 \quad ; \quad \text{for any } n\in\mathbb{N} \quad \begin{cases} u_{n+1} = u_n - 2 \\ v_n = 2^{u_n} \end{cases}$$
   a. Show that the sequence (v_n) is geometric. Determine its characteristic elements.
   b. We pose for all $n\in\mathbb{N}^*$: $S_n = v_0+v_1+\cdots+v_n$
   Set the following limit: $\displaystyle\lim_{n\mapsto+\infty} S_n = \frac{8}{3}$
2. We define the sequence (x_n) defined by:
   $$x_0 = 0 \quad ; \quad x_{n+1} = \frac{1}{2}x_n + \frac{3}{2} \quad \text{for all } n\in\mathbb{N}$$
   a. Show by recurrence that for any non-zero natural number n:
   $$x_n = \frac{3}{2} + \frac{3}{2^2} + \cdots + \frac{3}{2^n}$$
   b. Deduce a simple expression for x_n as a function of n.
   c. Determine the limit of the sequence (x_n).

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E.3424 Let $(u_n)_{n\in\mathbb{N}}$, be the sequence defined by the following recurrence relation:

$$\begin{cases} u_0 = 0 \quad ; \quad u_1 = 1 \\ u_{n+2} = \dfrac{4}{3}\cdot u_{n+1} - \dfrac{1}{3}\cdot u_n \quad \text{pour tout } n\in\mathbb{N} \end{cases}$$

1. Determine the exact value of the first five terms of the sequence (u_n).
2. We define the sequence $(v_n)_{n\in\mathbb{N}^*}$ defined by:

$$v_{n+1} = u_{n+1} - u_n \quad \text{for any natural number } n.$$

   a. Determine the first four terms of the sequence (v_n).
   b. Show that the sequence (v_n) is a geometric sequence; its characteristic elements will be specified.
3. By reasoning by recurrence, show that for any $n\in\mathbb{N}$, we have:
$$u_n = \frac{3}{2}\cdot\left[1-\left(\frac{1}{3}\right)^n\right]$$
4. Deduce the limit: $\lim\limits_{n\mapsto+\infty} u_n$.

8. *Sequences and probabilities*

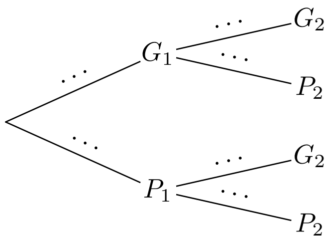
E.6202 Pierre and Claude are playing tennis. Both players have the same chance of winning the first game. Subsequently, when Pierre wins a game, the probability that he will win the next one is 0.7. And if he loses a game, the probability that he will lose the next one is 0.8.

Throughout the exercise, n is a non-zero natural number. We consider the events:

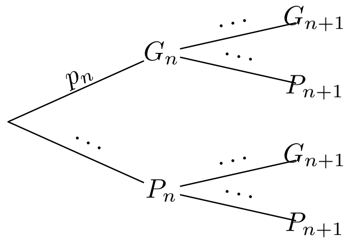
- G_n: "*Pierre wins the nth game*".
- P_n: "*Pierre loses the nth game*".

For any non-zero natural number n, we set: $p_n = \mathcal{P}(G_n)$.

1. Let's study the first two games:
   a. Complete the probability tree below:



   b. Determine the probability of event G_2.
   c. Given that Pierre won the second game, what is the probability that Claude won the first game?
2. Let's continue to study the games between Pierre and Claude:
   a. Complete the probability tree below:



   b. Deduce the relationship:
$$p_{n+1} = 0.5\times p_n + 0.2 \quad \text{for all } n\in\mathbb{N}^*$$

We consider the sequence (v_n) defined for all non-zero natural numbers n by the relationship: $v_n = p_n - \dfrac{2}{5}$.

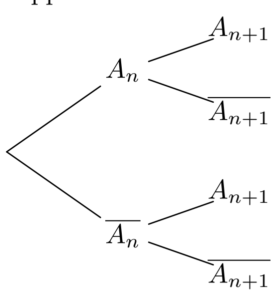
   c. Prove that the sequence (v_n) is a geometric sequence with common ratio 0.5
   d. Deduce an expression for the terms of the sequence (v_n), then for the terms of the sequence (p_n) in terms of n.
   e. Determine the limit of (p_n) when n tends to $+\infty$.
   f. How can we interpret the value of the limit of the sequence (p_n) in relation to the statement of the exercise?

E.6813 In a probabilized space $(\Omega;\mathcal{P})$. Consider a sequence of events (A_n) verifying the following relations:

$$\mathcal{P}(A_0) = 0.4 \quad ; \quad \begin{cases} \mathcal{P}_{A_n}(A_{n+1}) = 0{,}6 \\ \mathcal{P}_{\overline{A_n}}(A_{n+1}) = 0.4 \end{cases} \quad \text{for any } n\in\mathbb{N}$$

We note: $p_n = \mathcal{P}(A_n)$.

1. Complete the probability tree opposite.



2. Establish that:
$$p_{n+1} = 0.2\cdot p_n + 0.4$$
3. Demonstrate by recurrence that the terms of the sequence (p_n) admit for any natural number n:
$$p_n = -0.1\times 0.2^n + 0.5$$

9. *A little further: reasoning by recurrence*

E.3287 Consider the sequence (u_n) defined by:

$$u_n = \frac{n^{10}}{2^n} \quad \text{for any natural number } n.$$

We admit that for any natural number n greater than or equal to 16, we have:

$$\left(1+\frac{1}{n}\right)^{10} \leqslant 1.9$$

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Show, using reasoning by recurrence that for any natural number n greater than or equal to 16, we have the following frame:

$$0 \leqslant u_n \leqslant 0.95^{n-16} \cdot u_{16}$$

E.6206 Using reasoning by recurrence, establish that the equality below holds for any non-zero natural number n:

$$1^3 + 3^3 + \cdots + (2n-1)^3 = 2{\cdot}n^4 - n^2$$

E.3440 Show, using reasoning by recurrence, that for any non-zero natural number n and for any number x belonging to the interval $\big[-1\,;+\infty\big[$, we have the following property:

$$\big(1+x\big)^n \geqslant 1 + n{\cdot}x$$

E.3478 Prove by recurrence that for any non-zero natural number n, we have:

$$\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$$

E.6037 Consider the two sequences $\big(u_n\big)$ and $\big(v_n\big)$ defined for any integer $n \in \mathbb{N}^*$ by:

$$u_n = \sum_{k=1}^{n} k \quad ; \quad v_n = \sum_{k=1}^{n} k^2$$

1. Study the suite $\big(w_n\big)$ defined for any $n \in \mathbb{N}^*$ by:
$$w_n = \frac{v_n}{u_n}$$
2. Deduce an expression for the sequence $\big(v_n\big)$ as a function of n.

10. *Unclassified financial years*

E.6952 For each question, a statement is provided. Indicate whether it is true or false and justify your answer. Any answer without justification will not be considered.

- **Question 1**

  Consider the sequence $\big(u_n\big)$ defined by:
  $$u_0 = 1 \quad ; \quad u_{n+1} = 2{\cdot}u_n + 2{\cdot}n + 1$$
  Statement:

  The sequence $\big(u_n\big)$ is a geometric sequence.

- **Question 2**

  Consider the sequence $\big(u_n\big)$ defined by:
  $$u_0 = 0 \quad ; \quad u_{n+1} = u_n + 2{\cdot}n + 2$$
  Statement:

  For any natural number n, we have: $u_n = n^2 + n$

- **Question 3**

  Consider the geometric sequence $\big(u_n\big)$ with first term 4 and common ratio $\frac{1}{2}$.

  Consider the following algorithm:

```
u ← 4
n ← 0
S ← u
As long as  8−S⩾10^−2
      u ←  0.5× u
      n ←  n+1
      S ←  S+u
End while
```

  Assertion:

  At the end of the algorithm execution, the variable `n` has the value 10.

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