1. Any sequences

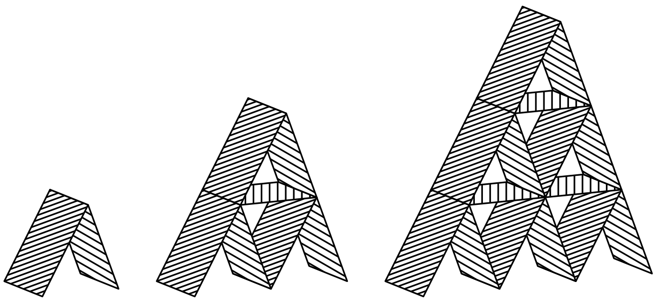
E.7716 For each question, determine the first four terms of the sequence $(u_n)_{n\in\mathbb{N}}$:

(a) $u_n = \dfrac{n+1}{n+2}$ (b) $u_{n+1} = 2{\cdot}u_n - 2$; $u_0 = 3$

(c) $u_n = \sqrt{n^2+n+1}$ (d) $u_{n+1} = 2{\cdot}u_n - 2$; $u_0 = 1$

(e) $u_n = \dfrac{n-2}{n+1}$ (f) $u_{n+1} = \dfrac{u_n - 2}{u_n + 1}$; $u_0 = 2$

E.4556 Consider the construction of a house of cards:

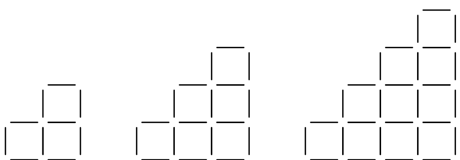


1^{er} étape $2^{\text{ième}}$ étape $3^{\text{ième}}$ étape

For any non-zero natural number $(n\in\mathbb{N}^*)$, we note u_n the number of cards needed to build the $n^{\text{ème}}$ step.
Thus, we have:

$u_1 = 2$; $u_2 = \dots$; $u_3 = \dots$; $u_4 = \dots$

E.7498 The figures below are constructed using small wooden sticks.



1^{er} étape 2^{e} étape 3^{e} étape

For n a strictly positive integer $(n\in\mathbb{N}^*)$, , we denote u_n the number of sticks needed for construction in step n. We therefore have:

$u_1 = 10$; $u_2 = \dots$; $u_3 = \dots$; $u_4 = \dots$

E.7598 Consider the sequence (u_n) defined for any integer n positive or zero $(n\in\mathbb{N})$ by:

$u_0 = 2$; $u_{n+1} = u_n + \dfrac{n}{n+1}$

Determine the first four terms of the sequence (u_n).

E.7492 Consider the following sequences of numbers:

(a) $a_0=4$; $a_1=7$; $a_2=10$; $a_3=13$; $a_4=16$ ...

(b) $b_0=1$; $b_1=-2$; $b_2=4$; $b_3=-8$; $b_4=16$ ...

(c) $c_0=1$; $c_1=3$; $c_2=5$; $c_3=7$; $c_4=9$ ...

(d) $d_0=16$; $d_1=8$; $d_2=4$; $d_3=2$; $d_4=1$ ...

Associate each of these sequences with one of the following relations, which defines the value of a term either in terms of the value of its predecessor or in terms of its rank:

(1) $u_{n+1} = u_n + 2$ (2) $u_n = 4 + 3{\cdot}n$

(3) $u_{n+1} = -2{\cdot}u_n$ (4) $u_n = 16\times\left(\dfrac{1}{2}\right)^n$

E.4557

(1) Here are some examples of number sequences:

(a) (2 ; 5 ; 8 ; 11 ; 14 ; ...)

(b) (2 ; 6 ; 18 ; 54 ; 162 ; ...)

(c) (6 ; -6 ; 6 ; -6 ; 6 ; ...)

(d) (1 ; 3 ; 7 ; 15 ; 31 ; ...)

For each of these number sequences, find the relationship that allows you to obtain a value based on the previous values.

(2) Here are some other examples of number sequences:

(a) (0 ; 2 ; 4 ; 6 ; 8 ; ...)

(b) (1 ; 6 ; 11 ; 16 ; 21 ; ...)

(c) (1 ; 2 ; 4 ; 8 ; 16 ; 32 ; ...)

(d) (1 ; $\sqrt{2}$; $\sqrt{3}$; 2 ; $\sqrt{5}$; $\sqrt{6}$; ...)

(e) $\left(2 \;;\; \dfrac{3}{2} \;;\; \dfrac{4}{3} \;;\; \dfrac{5}{4} \;;\; \dfrac{6}{5} \;;\; \dots \right)$

For each of these sequences of numbers, find the relationship that allows you to obtain a value based on its position in the sequence.

E.4559 Consider the sequences defined below by the value of their first term and a recurrence relation where the integer n denotes a positive or zero integer $(n\in\mathbb{N})$:

(a) $u_0 = 5$; $u_{n+1} = u_n + 2$ (b) $v_0 = 3$; $v_{n+1} = 2{\cdot}v_n$

(c) $w_0 = 2$; $w_{n+1} = -w_n$ (d) $x_0 = 4$; $x_{n+1} = 2{\cdot}x_n - 2$

(d) $y_0 = 1$; $y_1 = 1$; $y_{n+1} = y_n + y_{n-1}$

Determine the first five terms of each of these sequences.

E.4581 For each question, a sequence is defined where the rank n denotes a positive integer or zero $(n\in\mathbb{N})$. Determine the first five terms of the sequence (u_n):

a) $u_n = \dfrac{n+1}{n+2}$ b) $u_{n+1} = 2{\cdot}u_n - 2$; $u_0 = 3$

c) $u_n = \sqrt{n^2+n+1}$ d) $u_{n+1} = 2{\cdot}u_n - 2$; $u_0 = 1$

e) $u_n = \dfrac{n-2}{n+1}$ f) $u_{n+1} = \dfrac{u_n - 2}{u_n + 1}$; $u_0 = 2$

E.4604

1. Consider the sequence (u_n) whose rank term n, a positive or zero integer, is given by the relation:
$$u_n = -\frac{7}{4}\cdot\left[1-(-1)^n\right]+3$$
   a) Determine the first five terms of this sequence.
   b) What can be said about the value of the terms in the sequence (u_n)?

2. Consider the sequence (v_n) defined, for any positive or zero rank, by:
$$v_{n+1} = \frac{1-v_n}{1+v_n} \quad ; \quad v_0 = 3$$
   a) Determine the first five terms of the sequence (v_n).
   b) What do we notice?

E.7539 Consider the two algorithms below:

Algorithm 1
```
u ← 4
For i ranging from 1 to 5
    u ← u+3
End For
```

Algorithm 2
```
u ← 1
For i ranging from 1 to 4
    u ← 2× u+1
End For
```

For each algorithm, give the value contained in variable u after the algorithm has been executed.

E.7591

1. Determine the first four terms of the sequence (u_n) defined for any positive or zero integer $(n\in\mathbb{N})$:
$u_0=3 \quad ; \quad u_{n+1}=2{\cdot}u_n-2$

2. Determine the first four terms of the sequence (v_n) defined for any positive or zero integer $(n\in\mathbb{N})$:
$v_n = \dfrac{n+1}{2{\cdot}n+1}$

E.7597 Consider the sequence (u_n) defined for any integer n positive or zero $(n\in\mathbb{N})$ by:
$$u_0 = 3 \quad ; \quad u_{n+1} = 2{\cdot}u_n + \frac{1}{n+1}$$
Determine the first four terms of the sequence (u_n).

E.7592

1. Determine the first four terms of the sequence (u_n) defined for any positive or zero integer $(n\in\mathbb{N})$:
$u_0=4 \quad ; \quad u_{n+1}=3-2{\cdot}u_n$

2. Determine the first four terms of the sequence (v_n) defined for any positive or zero integer $(n\in\mathbb{N})$:
$v_n = \dfrac{2{\cdot}n-1}{n+3}$

E.4558 Consider the numerical sequences defined below, where their rank n is a positive integer or zero $(n\in\mathbb{N})$:

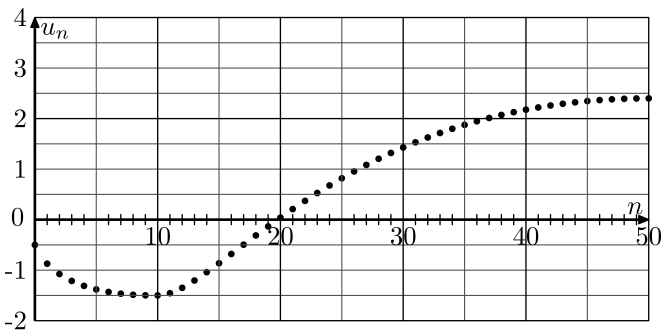
a) $u_n = 2n$ b) $v_n = 3n-4$

c) $w_n = n^2+3$ d) $x_n = 2^n$

Determine the first five terms of each of these sequences.

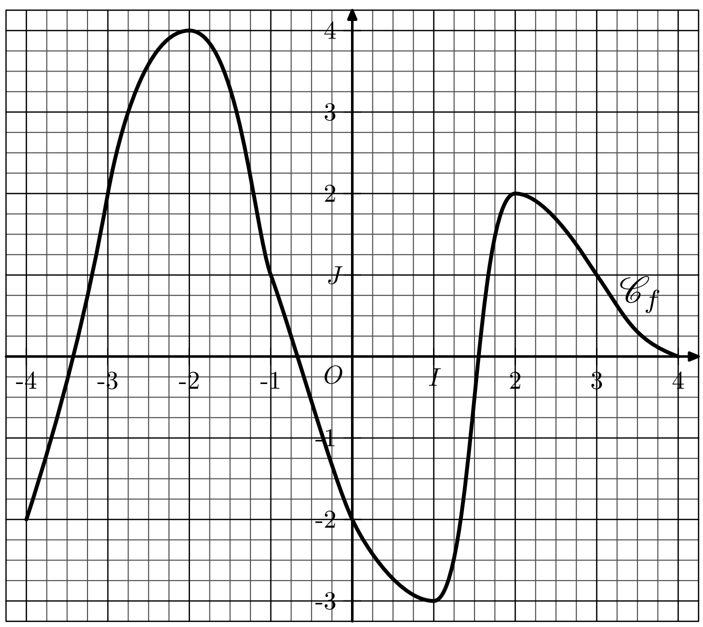
2. Any sequences and graphical reading

E.7750 Consider a sequence (u_n) defined for any natural number n positive or zero $(n\in\mathbb{N})$. In a reference frame are represented the points with coordinates $(n\,;u_n)$ for n between 0 and 50:



1. Give the exact values of u_0 and u_{10}.
2. Give approximate values of u_5, u_{30} and u_{40}.
3. What can be said about the values of the terms u_n as the value of n increases?

E.4584 In the plane provided with an orthonormal reference frame $(O\,;I\,;J)$, consider the representation $\mathscr{C}_f$ of a function f defined on the interval $[-4\,;4]$:



Consider the sequences $(u_n)_{n\in\mathbb{N}}$ and $(v_n)_{n\in\mathbb{N}}$ verifying the relations :

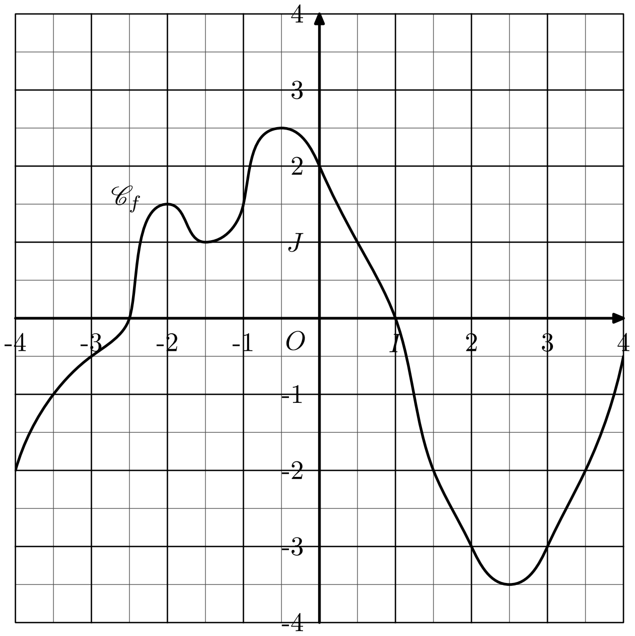
$$u_{n+1} = f(u_n) \quad ; \quad v_{n+1} = f(v_n)$$

verifying the following initial conditions :

$$u_0 = -1 \quad ; \quad v_0 = -4$$

Determine the first 100 terms of each of these two sequences.

E.4583 Consider the function f defined on $[-4\,;4]$ dont representative curve $\mathscr{C}_f$ est given below :



Consider the sequence $(u_n)_{n\in\mathbb{N}}$ defined by the relation

$$u_0 = 2 \quad \text{and} \quad u_{n+1} = f(u_n).$$

1. Show that the term u_1 est equals -3.
2. Justify the following equalities :

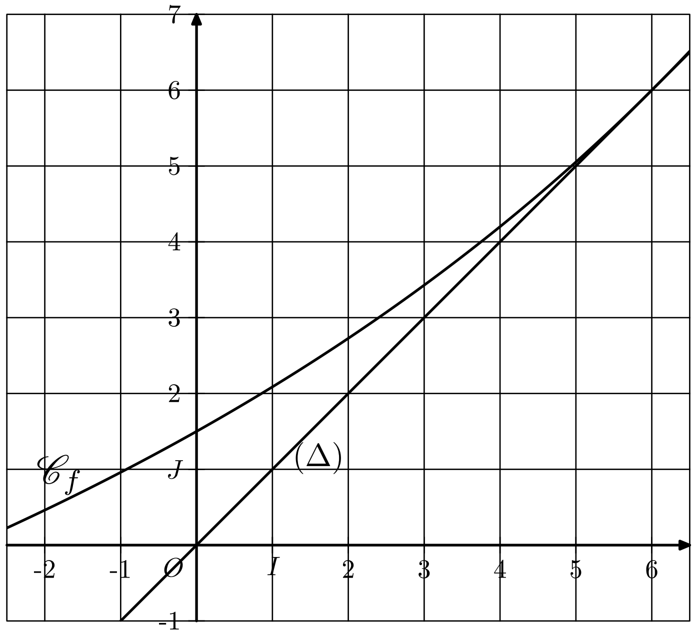
   a. $u_2 = -0.5$ b. $u_3 = 2.5$

3. Complete the following table :

n	0	1	2	3	4	5	6	7	8	9
u_n										

E.4610 In the plane provided with a $(O\,;I\,;J)$ orthonormal, consider the curve $\mathscr{C}_f$ representative of the function f defined by the relation :

$$f(x) = -\frac{12x+36}{x-24}$$



The line (Δ) is the first bisector of the plane.

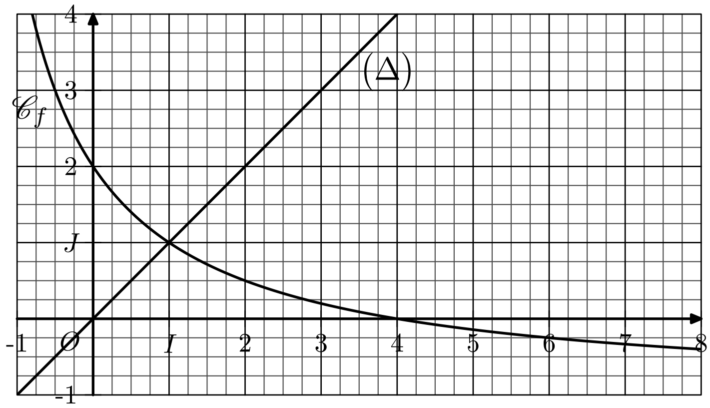
Consider the sequence $(u_n)_{n\in\mathbb{N}}$ defined by recurrence by :

$$u_{n+1} = f(u_n) \quad ; \quad u_0 = -2$$

1. Graphically, place on the x-axis the first five values of the sequence (u_n).
2. Determine, by calculation, the value of the first three terms of the sequence (u_n).

E.4609 In the plane provided with a $(O\,;I\,;J)$ orthonormal, consider the curve $\mathscr{C}_f$ of the function f whose image of a number x is defined by :

$$f(x) = \frac{-x+4}{x+2}$$



The line (Δ) is the first bisector of the plane.

Consider the sequence $(u_n)_{n\in\mathbb{N}}$ defined by the relation :

$$u_{n+1} = f(u_n) \quad ; \quad u_0 = 8$$

1. On the x-axis, show the values of the first six terms of the sequence.
2. Determine, by calculation, the first three terms of the sequence (u_n).

3. Sum of terms

E.7724

1. Consider the arithmetic sequence (u_n) with first term 4 and reason $\frac{2}{3}$
   a. Determine the first three terms of the sequence (u_n).
   b. Determine the rank of the sequence (u_n) having value 38.
   c. Determine the sum of terms:
   $S = u_0 + u_1 + \cdots + u_{99}$.
2. Consider the geometric sequence (v_n) of first term 4 and reason $\frac{2}{3}$
   a. Determine the first three terms of the sequence (v_n).
   b. Determine the sum of the terms:
   $S' = v_0 + v_1 + \cdots + v_{15}$.

E.7798

1. Consider the arithmetic sequence (u_n) with first term $\frac{1}{3}$ and reason $\frac{1}{2}$
   a. Determine the first three terms of the sequence (u_n).
   b. Determine the rank of the term u_n having value $\frac{77}{6}$.
   c. Determine the sum of terms:
   $S = u_0 + u_1 + \cdots + u_{99}$.
2. Consider the geometric sequence (v_n) of first term $\frac{1}{3}$ and reason $\frac{1}{2}$
   a. Determine the first three terms of the sequence (v_n).
   b. Determine the sum of the terms:
   $S' = v_0 + v_1 + \cdots + v_{15}$.

E.6547

1. Consider the suite $(u_n)_{n\in\mathbb{N}}$ arithmetic with first term 5 and reason 3.
   Determine the value of the sum S of the 100 first terms of this sequence.
2. Consider the geometric sequence $(v_n)_{n\in\mathbb{N}}$ of first term 3 and reason $\frac{1}{4}$.
   Determine the value of the sum S' of the first 100 terms of this sequence.

E.2453

1. Let $(u_n)_{n\in\mathbb{N}}$ be an arithmetic sequence of first term 2 and reason $\frac{3}{5}$, determine the value of the following sum:
   $S = u_5 + u_6 + \cdots + u_{21}$
2. Consider the following sum whose terms are those of a geometric sequence:
   $$S' = 16 + 24 + 36 + \cdots + \frac{3^{10}}{2^6}$$
   Determine the value of the sum S'.

E.5172 Consider the sequence (u_n) defined by:

$$u_0 = 0 \quad ; \quad u_{n+1} = -\frac{1}{2}\cdot u_n + 1 \quad \text{for all } n\in\mathbb{N}$$

1. Determine the first five terms of the sequence (u_n).
2. Consider the sequence (v_n) defined by:
   $$v_n = u_n - \frac{2}{3} \quad \text{for all } n\in\mathbb{N}$$
   a. Determine the first four terms of the sequence (v_n).
   b. Establish that for any natural number n, we have:
   $v_{n+1} = -\frac{1}{2}\cdot v_n$
   c. Give the nature and values of the characteristic elements of the sequence (v_n).
3. a. Determine the value of the sum S' defined by:
   $S' = v_0 + v_1 + \cdots + v_{14}$
   b. Determine the value of the sum S defined by:
   $S = u_0 + u_1 + \cdots + u_{14}$
4. a. Give the expression of the term v_n as a function of its rank n.
   b. Give the expression of the term u_n as a function of its rank n.

E.3017

1. Consider a sequence $(u_n)_{n\in\mathbb{N}}$ arithmetic such that:
   $$u_{13} = 7 \quad ; \quad u_{20} = \frac{35}{2}$$
   a. Justifying your approach, find the characteristic elements of this sequence.
   b. Deduce the value of the sum S defined by:
   $S = u_5 + u_6 + \cdots + u_{22}$
2. Consider a geometric sequence $(v_n)_{n\in\mathbb{N}}$ such that:
   $$v_9 = \frac{5^6}{7^4} \quad ; \quad v_{16} = \frac{5^{13}}{7^{11}}$$
   a. Justifying your approach, find the characteristic elements of this sequence.
   b. Deduce the value of the sum S' defined by:
   $S' = v_5 + v_6 + \cdots + v_{22}$

E.7607

1. Consider the sequence (u_n) geometric with first term 2 and reason 3.
   Determine the sum of the first 10 terms of the sequence (u_n).
2. Consider the sequence (v_n) geometric of first term 5 and reason $\frac{1}{2}$.
   Determine an expression for the sum S defined by:
   $S = v_0 + v_1 + v_2 + \cdots + v_{12}$
   Then give the value of S rounded to the nearest hundredth.

E.8471 The terms of each sum are the terms of a geometric sequence. Determine the value of these two sums:

1. $S_1 = 1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots + \dfrac{1}{1024}$
2. $S_2 = 2 + \sqrt{3} + \dfrac{3}{2} + \dfrac{3\sqrt{3}}{4} + \dfrac{9}{8}$

E.8467 Consider the sum S defined by:

$S = 5 + 2 - 1 - 4 - \cdots - 37$

Deduce the value of S.

E.7663 Consider the sum S defined by:

$S = 1 + \dfrac{5}{2} + 4 + \dfrac{11}{2} + \cdots + 100$

We admit that the terms of the sum S are the first successive terms of a sequence (u_n) arithmetic defined on $\mathbb{N}$.

1. Donner les éléments caractéristiques de la suite (u_n) et déterminer le rang du terme de la suite ayant 100 pour valeur.
2. Deduce the value of the sum S.

4. *Course - old program: Suites*

E.3291 The following results are assumed to be known:

Definition-proposition:

(1) two sequences (u_n) and (v_n) are said to be **adjacent** when: one is increasing, the other is decreasing, and $u_n - v_n$ tends toward 0 as n tends toward $+\infty$;

(2) if (u_n) and (v_n) are two adjacent sequences such that (u_n) is increasing and (v_n) is decreasing, then for any n belonging to $\mathbb{N}$, we have:
$u_n \leqslant v_n$;

(3) any increasing and bounded sequence is convergent; any decreasing and bounded sequence is convergent.

Then prove the following proposition:

"*Two adjacent sequences are convergent and have the same limit*".

E.3445 Consider the three sequences $(u_n)_{n\in\mathbb{N}}$, $(v_n)_{n\in\mathbb{N}}$, $(w_n)_{n\in\mathbb{N}}$ which verify the following conditions:

- The two sequences (v_n) and (w_n) converge to a real number ℓ.
- From a rank n_0 and for any integer n greater than n_0, we have:
  $vCOPY06u_n \leqslant w_n$

Show that the sequence (u_n) is convergent and, more precisely, has the following limit:

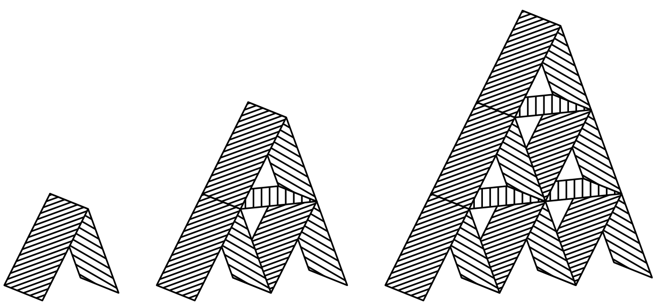
$\lim_{n\mapsto+\infty} u_n = \ell$

E.3475 Using the definition and the two properties below, show that if (u_n) and (v_n) are two adjacent sequences, then they are convergent and have the same limit.

- **Definition:**
  Two sequences are adjacent when one is increasing, the other is decreasing and the difference of the two converges to 0.
- **Property 1:**
  If two sequences (u_n) and (v_n) are adjacent with (u_n) increasing and (v_n) decreasing then for any natural number:
  $u_n \leqslant v_n$.
- **Property 2:**
  any increasing and majoring sequence converges; any decreasing and minoring sequence converges.

5. *Recalls on suites*

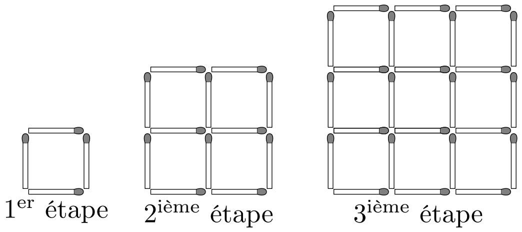
E.6124 Consider the construction of a house of cards:



1^{er} étape $\quad$ $2^{\text{ième}}$ étape $\quad$ $3^{\text{ième}}$ étape

How many cards are needed to complete the $4^{\text{ième}}$ stage of this construction? for the $5^{\text{ième}}$ stage?

E.6144 Consider the following constructions:



Note (u_n) the numerical sequence defined on $\mathbb{N}^*$ where u_n represents the number of matches required to construct the $n^{\text{ième}}$ step.

Which of the following definitions can be used to define the sequence (u_n) shown above:

(a) $\begin{cases} u_1 = 4 \\ u_{n+1} = 4\cdot u_n \end{cases}$ (b) $\begin{cases} u_1 = 4 \\ u_{n+1} = u_n + 4\cdot n \end{cases}$

(c) $\begin{cases} u_1 = 4 \\ u_{n+1} = u_n + 4\cdot(n+1) \end{cases}$ (d) $\begin{cases} u_1 = 4 \\ u_{n+1} = 4\cdot u_n + n \end{cases}$

E.6125 Consider the numerical sequences, below, defined on $\mathbb{N}$ by recurrence: i.e. according to their initial value and a relationship with the preceding terms.

1. $u_0=5$; $u_{n+1}=u_n+2$ for all $n\in\mathbb{N}$.
2. $v_0=3$; $v_{n+1}=2\cdot v_n$ for any $n\in\mathbb{N}$.
3. $w_0=2$; $w_{n+1}=-w_n$ for any $n\in\mathbb{N}$.
4. $x_0=4$; $x_{n+1}=2\cdot x_n-2$ for any $n\in\mathbb{N}$
5. $x_0=1$; $x_1=1$; $x_{n+1}=x_n+x_{n-1}$ for all $n\in\mathbb{N}$

Determine the first five terms of each of these sequences.

6. *Unclassified financial years*

E.4644 Consider the sequence (u_n) defined on $\mathbb{N}$ by the following recurrence relation:

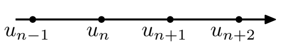
$u_0 = 0$; $u_{n+1} = u_n - 2n + 11$

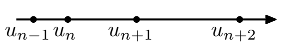
1. (a) Using the software of your choice, plot the scatter-plot associated with the first 15 terms of this sequence.
   (b) Make a conjecture as to the nature of the curve passing through these points.
2. (a) Determine the function f defined by a polynomial of the second degree, verifying the relations:
   $u_0 = f(0)$; $u_1 = f(1)$; $u_{11} = f(11)$
   (b) Give the reduced expression of the expression: $f(x+1) - f(x)$.
   (c) Establish the expression of the terms of the sequence (u_n) as a function of their rank n.

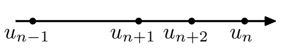
E.5105 Consider the sequence (u_n) defined by:

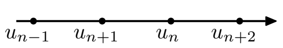
$u_0 = 1$; $u_{n+1} = -\dfrac{1}{2}\cdot u_n + 1$ for all $n\in\mathbb{N}$

1. Determine the first four terms of this sequence.
2. (a) Establish the following equality for all $n\in\mathbb{N}$:
   $u_{n+2} = \dfrac{1}{4}\cdot u_n + \dfrac{1}{2}$
   (b) Establish the following equality for all $n\in\mathbb{N}$:
   $u_{n+2} = \dfrac{u_n + u_{n+1}}{2}$
3. When representing four terms of the sequence (u_n) on a graduated line, which of the four propositions below would best represent them?

(a) 

(b) 

(c) 

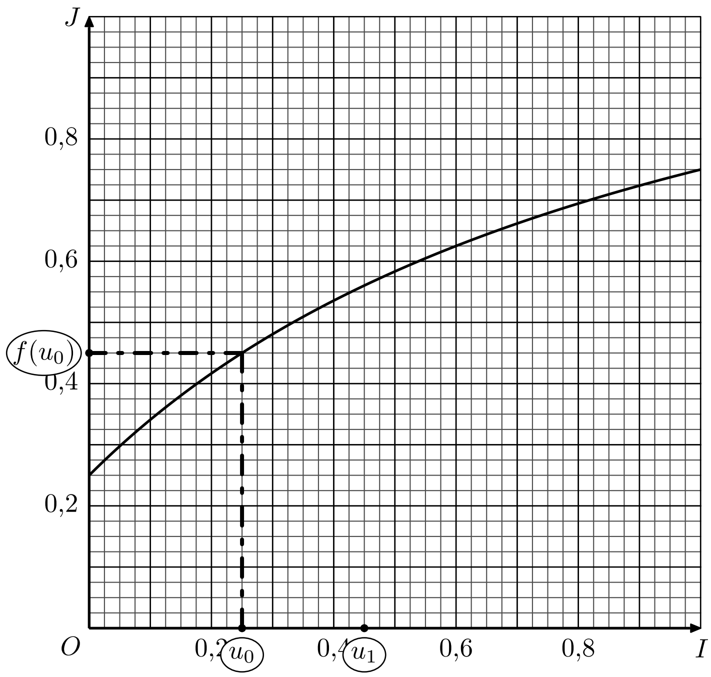
(d) 

E.6635 Consider the function f defined on $[0\,;1]$ by the relation:

$f(x) = \dfrac{5}{4} - \dfrac{1}{x+1}$

1. (a) Establish the following values:
   $f\Big(\dfrac{1}{4}\Big) = \dfrac{9}{20}$; $(f\circ f)\Big(\dfrac{1}{4}\Big) = \dfrac{65}{116}$
   (b) Determine the value of: $(f\circ f\circ f)\Big(\dfrac{1}{4}\Big)$

Below is the representative curve $\mathscr{C}_f$ of the function f in the orthonormal coordinate system $(O\,;I\,;J)$:



2. (a) Give the values rounded to the nearest thousandth of the following numbers:
   - $u_0 = \dfrac{1}{4} = \ldots\ldots$
   - $u_1 = \dfrac{9}{20} \approx \ldots\ldots$
   - $u_2 = \dfrac{65}{116} \approx \ldots\ldots$
   - $u_3 = \dfrac{441}{724} \approx \ldots\ldots$
   (b) Place the values u_2 and u_3 on the x-axis.

(c) Place the values $f(u_1)$, $f(u_2)$, and $f(u_3)$ on the y-axis.

(3) (a) Draw the line segment connecting the two points $A_1(u_1\,;0)$ and $B_1(0\,;f(u_0))$.
What is the nature of triangle OA_1B_1?

(b) For i ranging from 1 to 3, we define the points:
$A_i(u_i\,;0)$ and $B_i\big(0\,;f\big(u_{i-1}\big)\big)$
What are the natures of the triangles OA_iB_i?

(c) Place the numbers u_4 and u_5 on the x-axis defined by the relations:
$f(u_3)=u_4 \quad ; \quad f(u_4)=u_5$

(4) *Generation of the terms of the sequence:*

(a) Enter and execute this program in the programming language of your choice.

```
x ← 0.25
For i ranging from 0 to 100
    x ← 5/4 − 1/(x + 1)
End For
```

At the end of execution, what is the value of the variable x?

(b) What conjecture can be made about the terms of this sequence?

E.5107 Consider the sequence $\big(u_n\big)_{n\in\mathbb{N}}$ defined by:
$$u_0=5 \quad ; \quad u_n=\Big(1+\frac{2}{n}\Big)\cdot u_{n-1}+\frac{6}{n} \quad \text{pour } n\in\mathbb{N}^*.$$

(1) (a) Complete the following table:

n	0	1	2	3	4	5	6
u_n							

(b) Make a conjecture about the nature of the sequence $\big(d_n\big)$ defined by:
$d_n=u_{n+1}-u_n$

(2) We consider the sequence $\big(v_n\big)$ defined by:
$v_n=4n^2+12n+5 \quad$ for all $n\in\mathbb{N}^*$

(a) Give the simplified expression of the expression v_{n+1} as a function of n.

(b) Simplify the expression of: $\Big(1+\dfrac{2}{n+1}\Big)\cdot v_n+\dfrac{6}{n+1}$.
(We'll use the factorization:
$4x^3+24x^2+41x+21=(x+1)(4x^2+20x+21)$*)*

(c) What can we say about the sequences $\big(u_n\big)$ et $\big(v_n\big)$.