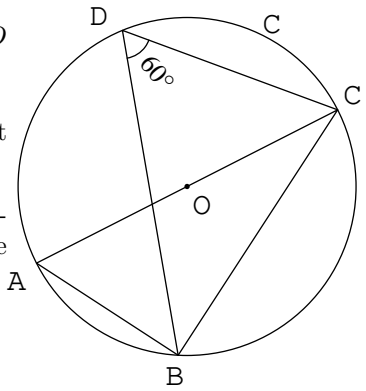
1. *Angles and trigonometry*

E.801

Let $\mathscr{C}$ be a circle of center O and radius $5\,cm$.
Points A, O, C are aligned.
D is a point of $\mathscr{C}$ such that $\widehat{BDC}=60^o$.

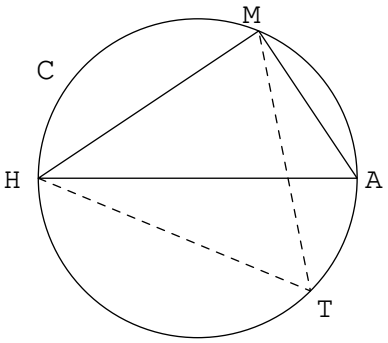
Determine the measure of segment $[BC]$ rounded to the nearest millimeter.



E.2493

On the figure opposite the measures are not respected.

Consider a circle $\mathscr{C}$ of diameter $HA=9\,cm$. Let M be a point on the circle $\mathscr{C}$ such that $MA=5.3\,cm$ and T another point on the circle $\mathscr{C}$.

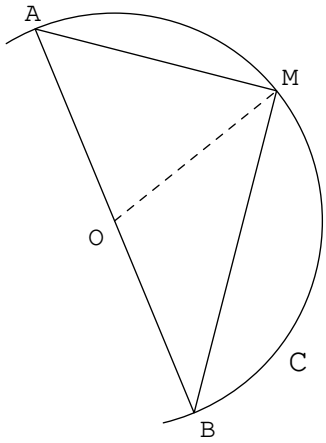


1. Justify that MAH is a right-angled triangle.
2. Calculate the measure, rounded to the nearest unit, of the angle $\widehat{MHA}$.
3. Determine the measure, rounded to the nearest unit, of the angle $\widehat{HTM}$.

E.2647

The figure opposite is not full-scale. It is not required to be reproduced.

$\mathscr{C}$ is a circle of center O and diameter $[AB]$ such that $AB=6\,cm$. M is a point on the circle such that $BM=4.8\,cm$.



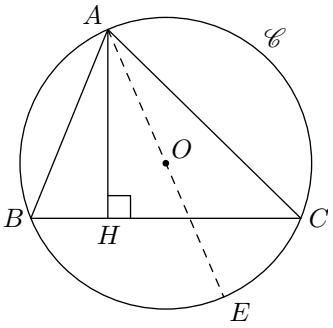
1. Demonstrate that the triangle ABM is right-angled at M.
2. Calculate the measure of the angle $\widehat{ABM}$, rounded to the degree.
3. Deduce the measure of the angle $\widehat{AOM}$, rounded to the degree.

E.4051

The figure below is not to be repeated on the copy. It is not given in full size.

A, B and C are three points on a circle $\mathscr{C}$ with center O.

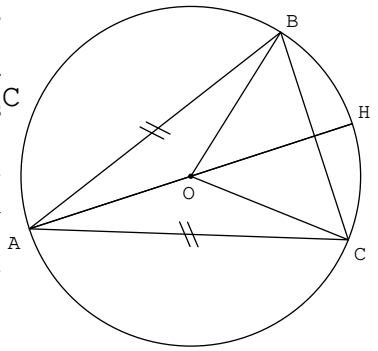
We know that $AB=3\,cm$. The height AH measures $2.5\,cm$. The diameter $[AE]$ is plotted.



1. What is the nature of the triangle ACE? Justify the answer.
2. Explain why the angles $\widehat{ABC}$ and $\widehat{AEC}$ are equal.
3. Using the triangle ABH, calculate the exact value of $\sin\widehat{ABH}$ and deduce the measure of the angle $\widehat{AEC}$ rounded to the degree.

E.798

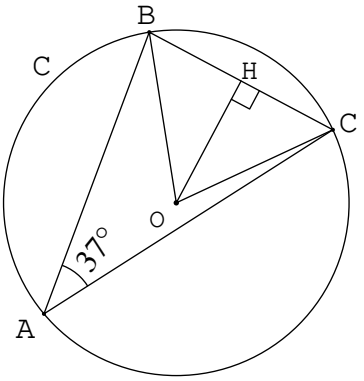
The figure opposite is not drawn to actual dimensions.
The circle $\mathscr{C}$ has as its center the point O and its radius is $5\,cm$.
The triangle ABC is inscribed in the circle $\mathscr{C}$ and such that $\widehat{BOC}=80^o$.
Note H the point of intersection of the straight line (AO) with the circle $\mathscr{C}$.



1. Show that the straight line (AO) is the perpendicular bisector of the segment $[BC]$.
2. Calculate the value of the angle $\widehat{OAB}$. Justify.
3. What is the nature of the triangle ABH? Justify.
4. Calculate the length of $[BH]$.

E.799

The figure opposite does not show the actual dimensions.
Let $\mathscr{C}$ be a circle of center O and radius $5\,km$. A, B and C are three points on the circle $\mathscr{C}$ such that $\widehat{BAC}=37^o$.
H is the foot of the height from O in the triangle ABC.

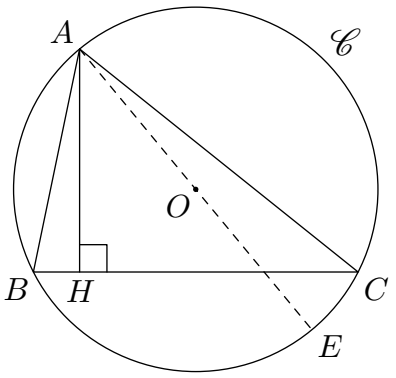


1. (a) Determine the measure of the angle $\widehat{BOC}$.
   (b) Calculate the value of the angle $\widehat{OBC}$.
2. Calculate the length of segment $[BH]$ rounded to the nearest decameter.

E.5406

The figure below is not to be repeated on the copy. It is not given in full size.

A, B and C are three points on a circle $\mathscr{C}$ *(see figure)*.
We know that $AB = 3\,cm$, the height $[AH]$ measures $2.5\,cm$.
We trace the diameter $[AE]$.



1. What is the nature of the triangle ACE? Justify the answer.
2. Explain why the angles $\widehat{ABC}$ and $\widehat{AEC}$ are equal.
3. Using the triangle ABH, calculate the exact value of $\sin\widehat{ABH}$ and deduce the measure of the angle $\widehat{AEC}$ rounded to the nearest degree.

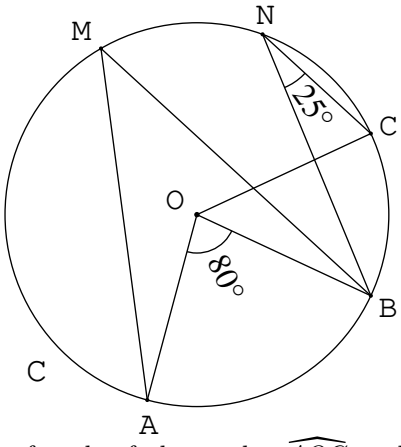
2. Angles and isosceles triangles

E.800

The points A, B, C, M and N belong to the circle $\mathscr{C}$ of center O verifying the measures:

$\widehat{AOB} = 80^o$; $\widehat{BNC} = 25^o$

1. Calculate the measure of $\widehat{AMB}$.
2. Calculate the measure of each angle of triangle BOC.

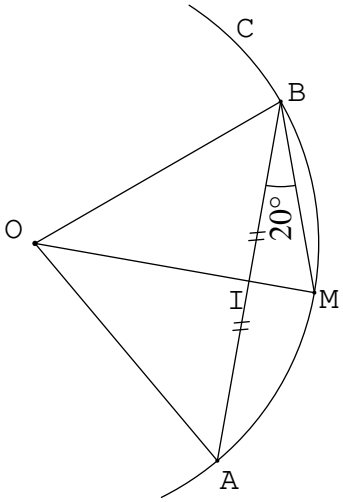


3. Deduce the measure of each of the angles $\widehat{AOC}$ and $\widehat{AMC}$.

E.802

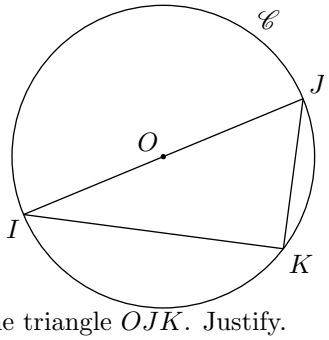
Let $\mathscr{C}$ be a circle of center O. A, B, M three points of the circle such that $\widehat{ABM} = 20^o$. Let I be the middle of segment $[AB]$.

Calculate the value of the angle $\widehat{AOB}$. Justify.



E.5405

The figure opposite is not full-scale; we do not ask you to reproduce it.
Consider a circle $\mathscr{C}$ with center O and diameter $8\,cm$.
I and J are two diametrically opposed points. K is a point of $\mathscr{C}$ such that $JK = 4\,cm$.

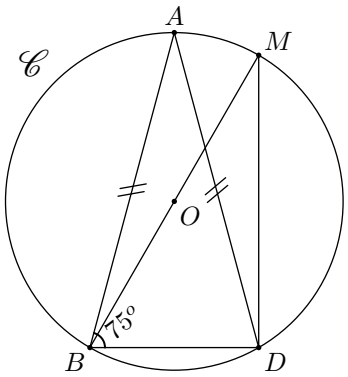


1. a) Specify the nature of the triangle OJK. Justify.
   b) Deduce the measure of the angle $\widehat{JIK}$. Justify your answer.
2. a) Specify the nature of the triangle IJK. Justify.
   b) Give the measurement, to the nearest millimetre, of segment $[IK]$.
3. We call R the image of K by symmetry of axis (IJ). Demonstrate that the quadrilateral $ROKJ$ is a rhombus.

3. Angles and right-angled triangles

E.4338 *Consider the figure below, which is not full-scale. It is not necessary to redo the figure.*

- ABD is an isosceles triangle at A such that: $\widehat{ABD} = 75^o$;
- $\mathscr{C}$ is the circumscribed circle of the triangle ABD;
- O is the center of the circle $\mathscr{C}$;
- $[BM]$ is a diameter of $\mathscr{C}$.

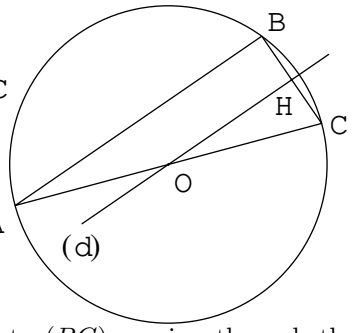


1. What is the nature of the triangle BMD? Justify the answer.
2. a. Calculate the measure of the angle $\widehat{BAD}$.
   b. Name an inscribed angle that intercepts the same arc of the circle $\mathscr{C}$ as the angle $\widehat{BMD}$.
   c. Justify that the angle $\widehat{BMD}$ measures 30^o.
3. We give: $BD = 5.6\,cm$; $BM = 11.2\,cm$
   Calculate DM. Round the result to the nearest tenth.

4. *Angles and thales*

E.805

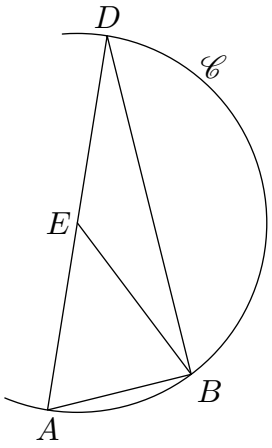
The figure opposite is not made to actual dimensions.
Let $\mathscr{C}$ be a circle of center O radius $6\,cm$.
Points A and C are diametrically opposed.
B is a point on the circle such that $BC = 3\,cm$.
The segment $[BC]$ measures $3\,cm$ in length.
The line (d) is perpendicular to (BC) passing through the point O. It intersects segment $[BC]$ at H.



1. What is the nature of the triangle ABC? Justify.
2. Determine the measure of the angle $\widehat{BAC}$.
3. a. Determine the measure of the angle $\widehat{BOC}$. Justify
   b. What is the nature of the triangle OBC? Justify
   c. Deduce the measure of the angle $\widehat{BOH}$
   d. Justify the fact that: $BH = 1.5\,cm$.
4. Calculate the value of OH to the nearest millimetre.

E.3928

On the figure opposite, which is not full-scale, we know that:
- $(\mathscr{C})$ est a circle with center E dont the diameter $[AD]$ mesure $9\,cm$.
- B is a point of circle $\mathscr{C}$ such that: $\widehat{AEB} = 46^o$



1. Make the figure respecting the given dimensions.
2. Show that the triangle ABD is a right-angled triangle.
3. Justify that: $\widehat{ADB} = 23^o$.
4. Calculate the length AB and specify its value rounded to the hundredth of cm.
5. Draw the line parallel to the line (AB) passing through E. It intersects segment $[BD]$ at point F. Place the point F.
6. Calculate the length EF and specify its value rounded to the tenth of cm.

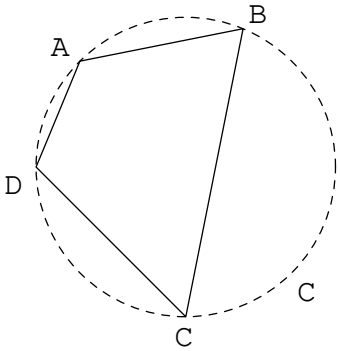
5. *Small and large bows*

E.2938

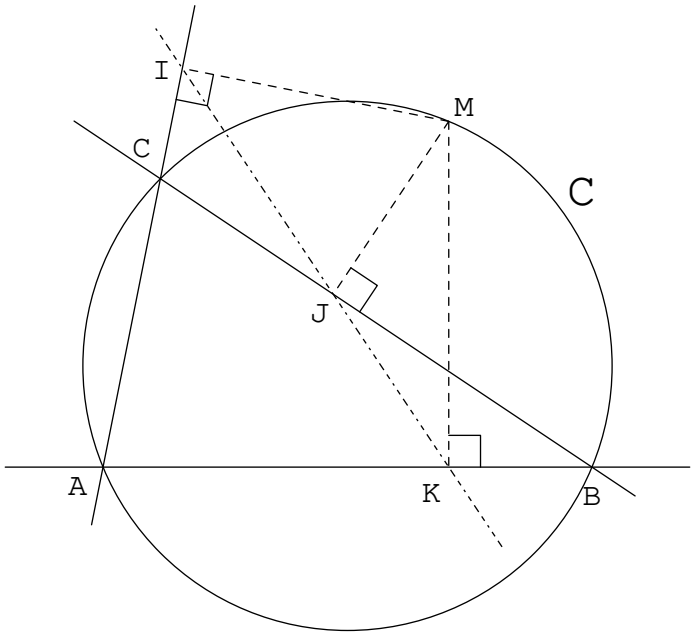
Consider a quadrilateral $ABCD$ inscribed in a circle $\mathscr{C}$.

Show that the opposite angles, in the quadrilateral $ABCD$, are angles of supplementary measures.

Hint: use the eight angles formed by the intersection of the two diagonals of the quadrilateral.



E.2939 In the plane, consider any triangle ABC and its circumscribing circle $\mathscr{C}$; the point M is any point on the circle $\mathscr{C}$; let I, J, K be the orthogonal projects of the point M onto the straight lines (AC), (BC), (AB), respectively.



The aim of this exercise is to show that the points I, J, K are aligned.

1 a Establish that the points I and J belong to the circle of diameter $[MC]$.

b Justify that the angles $\widehat{IJM}$ and $\widehat{ICM}$ have the same measure.

c Deduce the relationship: $\widehat{ACM} = 180 - \widehat{IJM}$

2 a Justify that the points B, J, K, M are cocyclic; Specify the nature of the circle to which they belong.

b Establish the equality: $\widehat{ABM} = 180 - \widehat{MJK}$
To demonstrate this relationship, we'll use the diagonals of the quadrilateral $JMBK$

3 The points A, B, C, M are cocyclic *(as in question 2 b)*; we accept the following relationship:
$\widehat{ABM} = 180 - \widehat{ACM}$

Deduce that the points I, J, K are aligned

The straight line passing through these three points is called Simson's straight line.

6. *A little of everything*

E.2492 ⚠ Demonstrate, for each of the three figures below, that the triangle ABC is a right-angled triangle using the information provided.

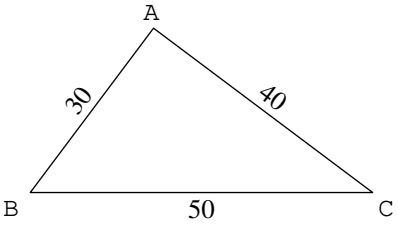


Figure n°1

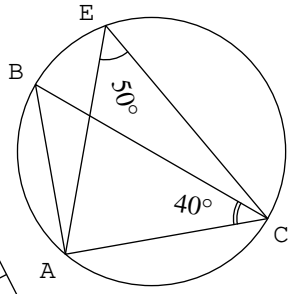


Figure n°3

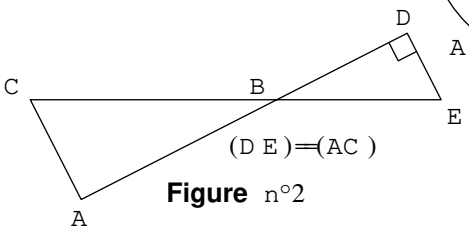


Figure n°2

E.3274 ⚠

$(AC)//(DE)$

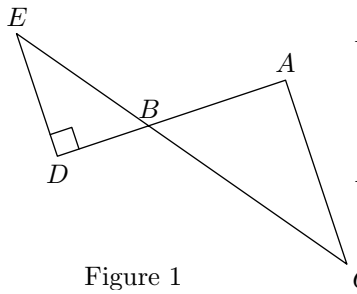


Figure 1

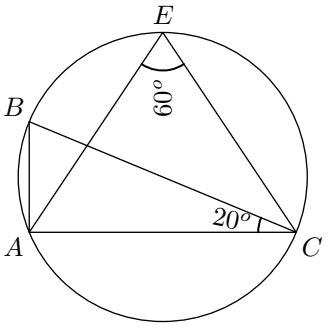


Figure 2

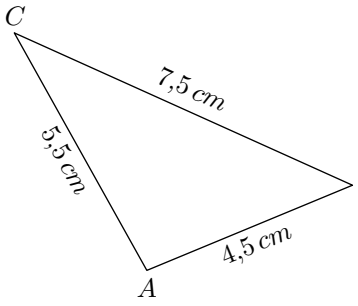
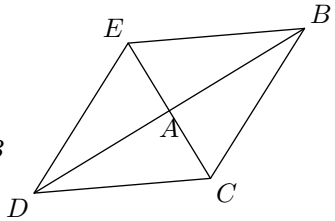


Figure 3



$BCDE$ est un losange de centre A

Figure 4

Complete the table below:

	Figure 1	Figure 2	Figure 3	Figure 4
Is triangle ABC a right triangle in A?	☐ Yes ☐ No	☐ Yes ☐ No	☐ Yes ☐ No	☐ Yes ☐ No
Number(s) of the property or properties that prove this				

List of properties:

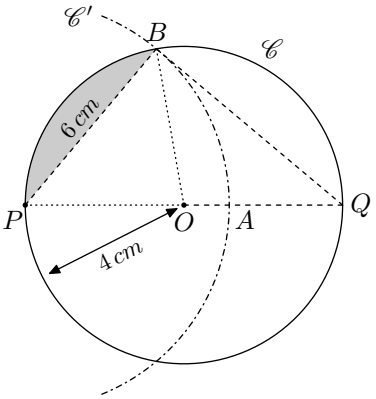
1. If a quadrilateral is a rhombus, then its diagonals have the same midpoint and are perpendicular.
2. If two lines are perpendicular to the same third line, then they are parallel to each other.
3. If, in a triangle, the square of the length of the longest side is not equal to the sum of the squares of the lengths of the other two sides, then the triangle is not a right triangle.
4. In a triangle, the sum of the measures of the three angles is equal to 180^o.
5. If two lines are parallel and a third line is perpendicular to one of them, then it is perpendicular to the other.
6. If a quadrilateral has four sides of equal length, then it is a rhombus.
7. If two angles inscribed in a circle intercept the same arc, then they have the same measure.
8. If in a triangle, the square of the length of the longest side is equal to the sum of the squares of the lengths of the other sides, then this triangle is a right triangle and the right angle is the angle opposite the longest side.

7. *Unclassified financial years*

E.5989

The circle $\mathscr{C}$ has center O and radius $4\,cm$. Note $[PQ]$ a diameter of the circle $\mathscr{C}$.

We construct the circle $\mathscr{C}'$ of center P and radius $6\,cm$. Note A the point of intersection of the circle $\mathscr{C}'$ and the segment $[PQ]$ and B one of the of the intersection points of the two circles $\mathscr{C}$ and $\mathscr{C}'$.



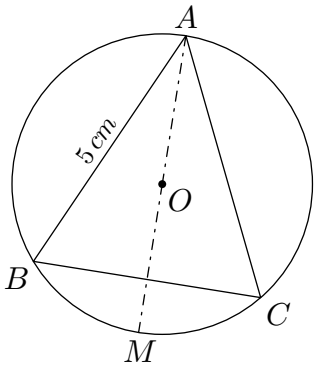
The aim of the exercise is to determine an approximate value for the area of the shaded part.

We will work with rounded measurements:

1. a) Determine the area of the triangle BPQ.
   b) Deduce the area of triangle POB.
2. a) Determine the measure, rounded to a tenth of a degree, of the angle $\widehat{PQB}$.
   b) Deduce the area, rounded to the tenth of cm^2, of the angular sector of the circle $\mathscr{C}$ defined by the arc $\widehat{PB}$.
3. Deduce a measure, rounded to the tenth of cm^2, of the shaded part.

E.6058

Consider an isosceles triangle ABC at A such that angle $\widehat{BAC}$ measures 50^o and AB is equal to $5\,cm$.
Let O be the center of the circle circumscribed around triangle ABC. The line (OA) intersects this circle, denoted $(\mathscr{C})$, at another point M.



1. What is the measure of angle $\widehat{BAM}$?
   No justification is required.
2. What is the nature of triangle BAM?
   Justify your answer.
3. Calculate the length AM and round it to the nearest tenth.
4. The line (BO) intersects the circle $\mathscr{C}$ at another point K. What is the measure of angle $\widehat{BKC}$?
   Justify your answer.